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INERTIAL GUIDANCE FOR LUNAR PROBES

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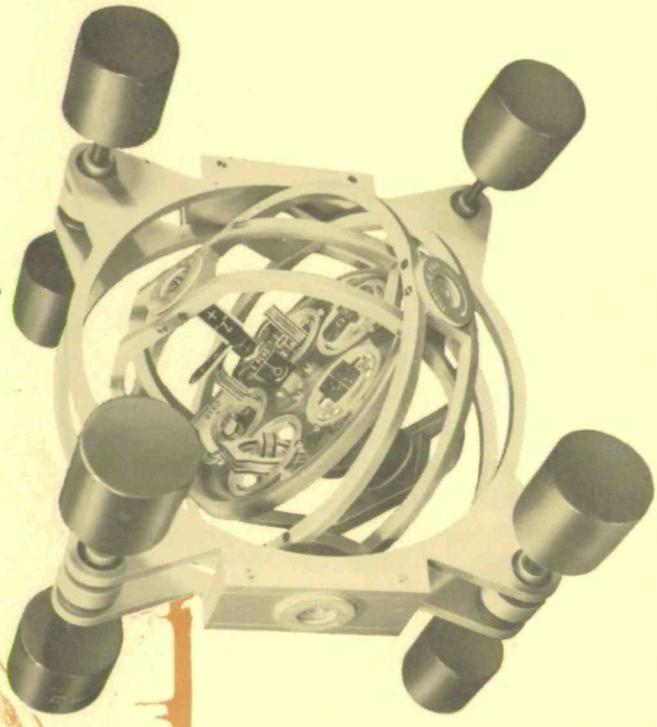

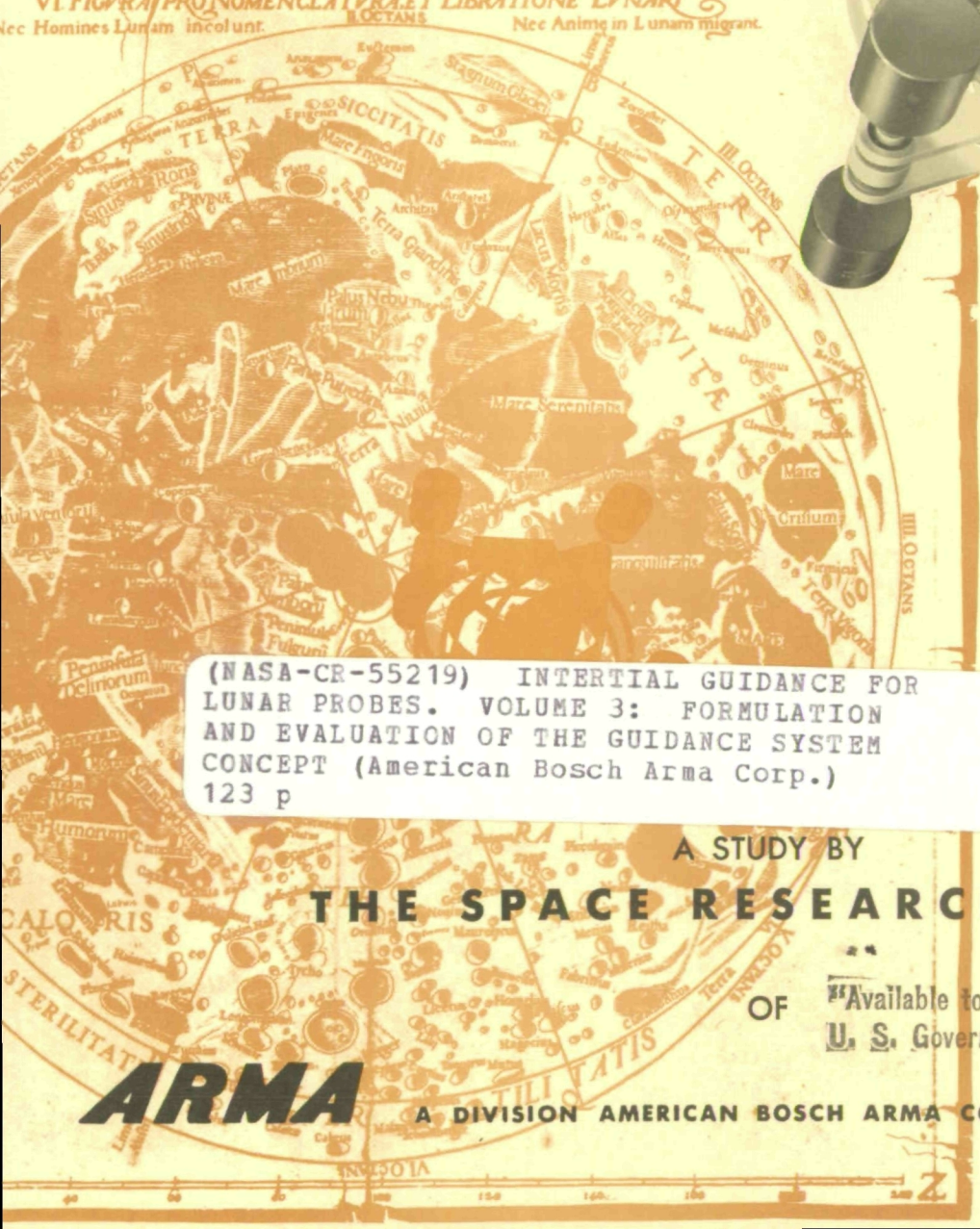

VOLUME III

NASA ~~CR~~ 55219

(NASA-CR-55219) INTERTIAL GUIDANCE FOR LUNAR PROBES. VOLUME 3: FORMULATION AND EVALUATION OF THE GUIDANCE SYSTEM CONCEPT (American Bosch Arma Corp.) 123 p
N74-74067
Unclas
00/99 38689

A STUDY BY

THE SPACE RESEARCH STAFF

OF

"Available to U.S. Government Agencies and U.S. Government Contractors Only."

ARMA
A DIVISION AMERICAN BOSCH ARMA CORPORATION GARDEN CITY, N.Y.

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on the cover ---

An inertial platform symbolically casts its shadow against the moon. In 1651 the Jesuit astronomer Giovanni Riccioli devised this lunar map which identifies outstanding features of the moon's surface.

Riccioli's nomenclature persists to the present day.

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VOLUME III 3)

FORMULATION AND EVALUATION OF THE GUIDANCE SYSTEM CONCEPT (C)

INERTIAL GUIDANCE FOR LUNAR PROBES. VOLUME 3 ...

This work was performed for the Jet Propulsion Laboratory, California Institute of Technology, sponsored by the National Aeronautics and Space Administration under Contract NASw 6

JPL Contract Number L-46075

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GARDEN CITY, NEW YORK, N.Y.

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PREFACE

The primary function of an injection guidance system is to steer the missile while in powered flight and terminate thrust at an appropriate time so that the subsequent free ballistic trajectory of the vehicle will possess certain desired characteristics which have been selected prior to the flight. In particular, the present study emphasizes the lunar interception problem. Therefore, we require that the injection guidance system control the powered flight of the missile and terminate thrust so that the ballistic trajectory of the vehicle will intercept the moon. In this volume, we are concerned with the formulation and evaluation of a guidance system concept which accomplishes this goal. The considerations which must be examined include the formulation of a guidance philosophy, the derivation of a mathematical model for implementing this philosophy, the approach to be taken in dealing with operational problems, the use of simulation techniques for evaluating the recommended system.

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SECTION 1

INTRODUCTION

In this study, we are concerned with the application of inertial systems for the injection guidance of space vehicles. As a typical mission, attention has been focused on lunar impact vehicles. The system developed, however, is capable of guiding space vehicles on a wide variety of missions such as satellite orbits, interplanetary injection, etc. In this volume, we are concerned primarily with the theoretical model of the guidance system rather than with its physical realization in terms of specific equipment. (A description and performance analysis of the guidance system equipment will be found in Volume IV of this report.) Although our concern here is primarily with the system concept, detailed attention has been devoted to assure that the recommended guidance system is compatible with the conditions imposed by vehicles anticipated for operational use during the next 2-10 year period.

The present volume starts with some qualitative considerations which served as guidelines to the formulation of the system. Based on these qualitative considerations, the important characteristics of the system are developed. This includes the manner in which navigation data (position, velocity, time) are processed in the system to arrive at guidance signals, and the way in which these guidance signals control the powered flight of the missile.

It is felt that, in a study of this type, the only method of demonstrating the feasibility of the recommended approach (short of actual flight test) is through careful quantitative analysis. Hence, a rather detailed discussion of many simulated trajectories of missiles under control of the guidance system is included.

A careful attempt was made in developing the simulation techniques for this study to include those characteristics of the vehicle which influence the performance of the guidance system. These vehicle characteristics (thrust profile, deviations in engine performance, operational considerations leading to delayed launch, etc.) have been selected to be typical of the conditions anticipated in actual vehicles. Although the emphasis in the present study has been on lunar probe missions, the library of simulation programs which has been developed provides sufficient flexibility for the study of many other space missions.

SECTION 2

GENERAL GUIDANCE PHILOSOPHY

The primary function of an injection guidance system is to steer the missile while in powered flight and terminate thrust at an appropriate time so that the subsequent free ballistic trajectory of the vehicle will possess certain desired characteristics which have been selected prior to the flight. For example, an ICBM trajectory must have the characteristic that it will intercept a pre-assigned point on the Earth's surface; a satellite trajectory must have certain desired orbital characteristics (period, eccentricity, etc). In particular, the present study emphasizes the lunar interception mission. Therefore, we require that the injection guidance system control the powered flight of the missile and terminate thrust so that the ballistic trajectory of the vehicle will intercept the moon.

The entire future ballistic motion of a space vehicle is determined by its state at the instant of injection (thrust termination). This state can be completely characterized by the seven injection variables $x_i, Y_i, z_i, \dot{x}_i, \dot{Y}_i, \dot{z}_i, t_i$ where x_i, Y_i, z_i are the injection position coordinates, $\dot{x}_i, \dot{Y}_i, \dot{z}_i$ are the injection velocity components (both position and velocity being described in an appropriate coordinate system to be specified later) and t_i is the injection time.

An arbitrarily selected set of the seven injection conditions will not, in general, result in the vehicle's interception of the moon, but will result in a non-zero miss. The miss at the moon can be characterized in a variety of ways, but these different characterizations all have a common attribute, i.e., the miss at the moon can be described in terms of two components. (NOTE: This is in close analogy to a surface-to-surface missile in which the miss at the target can also be expressed in terms of two components, e.g., down-range miss and cross-range miss.)

The two components of the miss at the moon are referred to as ϵ_V and ϵ_H . A precise definition of these miss components is deferred until later in this volume. For the present, it is sufficient to note that the miss components ϵ_V and ϵ_H are both well defined functions of the injection conditions. Stated symbolically:

$$\epsilon_V = f_1(x_i, Y_i, z_i, \dot{x}_i, \dot{Y}_i, \dot{z}_i, t_i) \qquad (2\text{-}1)$$

$$\epsilon_H = f_2(x_i, Y_i, Z_i, \dot{x}_i, \dot{Y}_i, \dot{z}_i, t_i) \qquad (2\text{-}2)$$

If the injection conditions are to result in a hit, then we require that at injection:

$$\left.\begin{aligned} f_1(x_i, Y_i, Z_i, \dot{x}_i, \dot{Y}_i, \dot{z}_i, t_i) = 0 \\ f_2(x_i, Y_i, Z_i, \dot{x}_i, \dot{Y}_i, \dot{z}_i, t_i) = 0 \end{aligned}\right\} \qquad (2\text{-}3)$$

Equation (2-3) implies the following important statement: If we require that the vehicle impact the moon, then the 7 injection variables must satisfy 2 relationships, i.e., only 5 injection variables can be selected independently; the remaining 2 variables are dependent functions of the first 5.

The primary function of the injection guidance system, then, is to control the powered flight of the missile so that at injection the two relationships (2-3) are simultaneously satisfied. Clearly, if the powered flight is to terminate with 2 relationships satisfied, then a minimum of two control parameters of the missile must be available to the guidance system during the powered flight. The control parameters of the missile which are normally available are:

....pitch steering

....azimuth steering

....thrust control (cut-off)

Concerning thrust control, it should be noted that while future vehicles may incorporate continuously variable thrust via some throttling arrangement, we assume in this study that the only form of thrust control available in the time period of interest is a choice of the moment at which thrust is to be terminated.

Since only two control parameters are required to impact the moon, it is necessary to decide which of the three available parameters shall not be called upon to satisfy the impact conditions. Pitch steering has been eliminated for the following reasons: In laying out missile trajectories, there are many important trajectory considerations pertinent to optimum fuel utilization, structural problems, and mission requirements which are closely related to the manner in which the missile is steered in pitch.

Typical of these important considerations are the following:

....The shaping of a trajectory to allow a given engine to place maximum payload in space is almost completely governed by the pitch steering of the vehicle.

....In the early portion of the powered flight while the vehicle is still in the atmosphere, strucutral requirements demand that the angle of attack be held within rather narrow limits. This is achieved by causing the vehicle to fly a zero-lift turn, which is accomplished primarily by appropriate steering of the missile in pitch.

....For certain lunar missions, the injections guidance system will be required to satisfy other conditions in addition to simply insuring impact at a certain point. A soft-landing lunar vehicle is an excellent case in point. For such a vehicle, the velocity of approach to the moon must be "killed" by braking rockets. Therefore, the braking rockets must have excess fuel capable of absorbing any uncertainty in the approach velocity. The larger this uncertainty, the larger will be the amount of payload sacrificed to excess braking fuel. Hence, it will be necessary to pre-determine at injection the approach velocity with minimum uncertainty. This pre-determination means that three relationships between the injection variables must be satisfied at the moment of injection. This can only be accomplished through the use of pitch steering.

From the above general discussion, we draw the following conclusion: There are so many important considerations related to pitch steering which are completely independent of the pure lunar interception problem, that pitch steering should be completely freed from any involvement with interception in order to leave it available for control of these other considerations. Thus, for example, a system concept which employs pitch steering solely to ease the interception guidance requirements placed on azimuth steering and thrust cut-off does not permit the satisfaction of any of the more sophisticated requirements.

There remains the problem of what shall be done with pitch steering for the purposes of the present study. In order to limit the scope of work the following decision was made: The pitch attitude of the missile shall be a pre-programmed function of time. The pitch attitude program shall be such that for a nominal missile the powered trajectory shall be a zero-lift turn.

With the establishment of the feasibility of achieving a precise lunar impact through the use of azimuth steering and thrust cutoff, it becomes a straightforward additional task to study the optimum manner of using pitch guidance for the satisfaction of additional conditions. (In fact, the Arma Space Research Group will investigate this problem upon completion of the present study.)

To summarize the main points of this section:

....We shall be concerned only with the lunar interception problem.

....Lunar interception requires that the injection variables satisfy two relationships.

....The control parameters selected to insure that these two relationships are satisfied at injection are thrust cut-off and azimuth steering.

....Pitch steering shall be controlled via a pre-programmed pitch profile.

....More complex pitch steering schemes should be studied as a means of satisfying important mission requirements in addition to interception.

SECTION 3

TECHNICAL APPROACH TO THE GUIDANCE SYSTEM FORMULATION

3.1 General Remarks

Having established some general guidelines to be followed in formulating the guidance concept for the lunar interception mission, we proceed now to examine the manner in which these guidelines are to be carried out. In the present section, we shall outline the guidance system's approach from a conceptual viewpoint, and in section 4 we shall present a more detailed discussion together with an evaluation of the system based on simulated flight data. In the present section, the major topics to be covered are:

- ....definition of guidance coordinate system
- ....definition of vehicle miss at the moon
- ....description of "velocity-to-be-gained" guidance approach
- ....method of controlling missile flight
- ....outline of guidance computations performed by airborne computer

3.2 Guidance Coordinate System

As described earlier, the vehicle position and velocity are expressed in terms of rectangular components $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}, t$. The coordinate system used is fixed in inertial space with the origin located at the launch point in inertial space and the axes aligned as follows:

- ....The Z axis is parallel to the pitch axis of a nominal missile flying the nominal trajectory (Y positive to the left when facing downrange).
- ....The X axis forms a right-handed system with Y and Z .

It should be noted that not only are the guidance computations made in this coordinate system, but also the physical accelerometers of the inertial platform are aligned with these axes. This is done to realize certain advantages related to platform pre-flight alignment and inertial component performance which will be discussed more fully in Volume IV of this report.

3.3 Definition of Miss Used in Deriving Guidance Equations

Volume II of this report has already considered at great length the properties of lunar probe ballistic trajectories. Therefore, only those points which are of direct interest will be reiterated here.

It was indicated earlier that in order to insure lunar interception, the two components of the miss, ϵ_V and ϵ_H, expressed as functions of the 7 injection variables, must vanish simultaneously, that is:

$$\left.\begin{aligned} \epsilon_V &= f_1(x_i, Y_i, Z_i, \dot{x}_i, \dot{Y}_i, \dot{z}_i, t_i) = 0 \\ \epsilon_H &= f_2(x_i, Y_i, z_i, \dot{x}_i, \dot{Y}_i, \dot{z}_i, t_i) = 0 \end{aligned}\right\} \qquad (3\text{-}1)$$

We now proceed to define the precise geometrical significance which has been assigned to the miss components ϵ_V and ϵ_H, which to this point have been abstract measures of the miss. In this connection, figure 3-1 is of value. We shall first define the miss components and then justify the choice of the definition.

First, it is assumed that the moon's gravitational field does not influence the motion of the vehicle. Next, consider an Earth centered sphere whose radius equals the distance from the center of the Earth to the center of the moon. At the instant the vehicle pierces this sphere, draw the vector $\bar{\epsilon}$ from the center of the moon to the vehicle. This vector is known as the chordal miss. The vector $\bar{\epsilon}$ is resolved into two components; ϵ_V, the vertical miss, lies in the vehicle's orbital plane, and ϵ_H, the horizontal miss, is perpendicular to the vehicle's orbital plane.

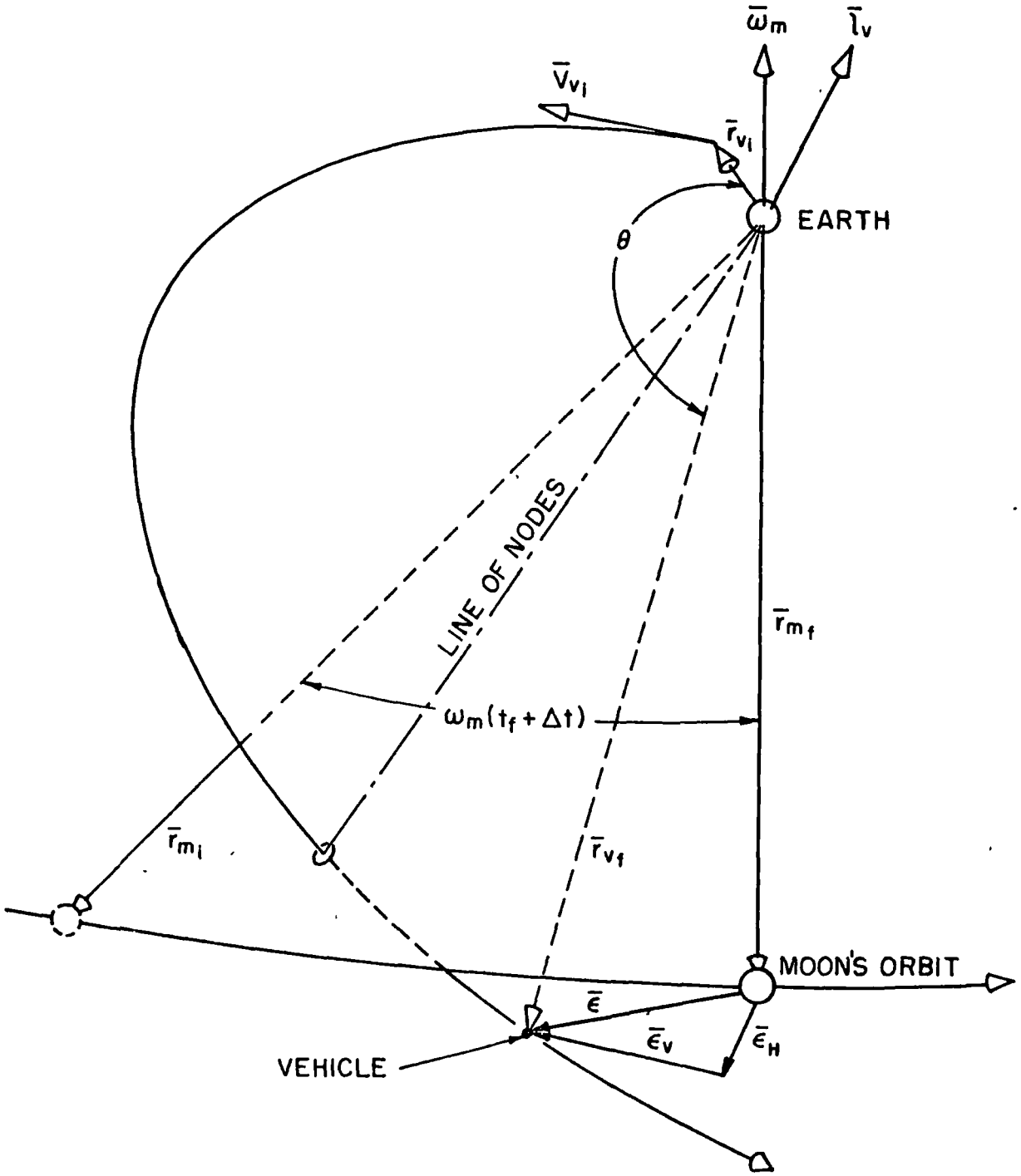

Fig 3-1 Geometry of the Lunar Interception Problem

The basis for this choice of the miss is as follows: First, the omission of the effect of lunar gravitation when considering the interception mission has been discussed at length in Volume II, where it was concluded that a set of injection conditions which would result in a normal impact if the moon had no gravitational field, will in fact result in a normal impact when the effect of lunar gravity is included. The primary effect that lunar gravitation has on a direct impact trajectory is to slightly shorten the ballistic flight time, a typical case being a shortening by about 10 minutes. An intuitive explanation of the fact that excluding lunar gravitation does not produce any interception error is given by noting that the effect of lunar gravitation is always to reduce the miss. Hence if the miss ignoring moon's gravity is zero, it certainly will be zero if lunar gravity is included. Thus, the omission of the lunar gravitation effect allows appreciable simplication of analysis without compromising the validity of the results.

The chordal miss, $\bar{\epsilon}$, (see Figure 3-1) also was chosen because of the simplifications in analysis which it allows. This simplification stems primarily from the fact that $\bar{\epsilon}$ is always evaluated when the vehicle is at a fixed radius from the Earth's center, specifically, the radius of the moon's orbit. Hence, we can allow the ballistic trajectory to always be computed directly to this radius when performing analyses and digital simulations related to formulation of the guidance concept.

What about other possible definitions of the miss components? More specifically, it is clear that our definition of miss at the moon has been selected completely on the basis of convenience, i.e., to achieve maximum simplification of analysis. Is this a legitimate basis for selection? Convenience and simplicity as a basis of choice can be justified as follows: Equation (3-1) states that interception requires two functions (f_1 and f_2) of the injection conditions to vanish at the moment of injection. A different physical definition of the miss components would result in two new functions (f_1' and f_2') having different functional dependence on the seven injection variables.

However, the primed and unprimed f's have an important characteristic in common: If the injection conditions are such as to result in a hit, then $f_1 = f_1' = 0$, and $f_2 = f_2' = 0$. Hence any legitimate miss components which provide a measure of the miss, i.e., which vanish when the interception error is zero, can serve as a basis for deriving an interception guidance system concept. Thus, the choice made in the present study is reasonable because of the easing of computational problems which it introduces.

Of course, there are other miss definitions which have important physical significance. Typical are:

....the distance of closest approach of the vehicle to the center of the moon ignoring lunar gravity (the so-called unfocused miss or $\bar{b}$ vector)

....the distance of closest approach of the vehicle to the center of the moon including lunar gravity.

These two quantities because of their important physical significance have been calculated in the present study in evaluating the guidance concept, but played no role in the derivation of the guidance concept. A mathematical description of the miss calculation technique and the corresponding IBM 704 program is given in Volume V, Section 3.

3.4 Velocity-to-be-Gained Approach Used to Satisfy the Two Interception Relations

As explained previously, the injection condition for a lunar interception requires that the two miss components ϵ_V and ϵ_H, expressed as functions of the injection variables must both vanish at the moment of injection:

$$\left.\begin{aligned} \epsilon_V &= f_1(x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i, t_i) = 0 \\ \epsilon_H &= f_2(x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i, t_i) = 0 \end{aligned}\right\} \qquad (3\text{-}2)$$

It was indicated earlier that if 2 relations are to be satisfied by the 7 variables, then only 5 variables can be chosen independently, the remaining 2 being dependent functions of the first 5. In the system under study, the 5 independently chosen variables are X , Y , Z , $\dot{Z}$, t . The 2 dependent variables are $\dot{X}$ and $\dot{Y}$ and are found in principle by simultaneous solution of the 2 equations (3-2). The results of this solution are indicated by the symbols $\dot{X}_R$ and $\dot{Y}_R$, the subscript R indicating that these are the values required for impact of the moon, assuming the other 5 variables given. The simultaneous solution of the 2 equations (3-2) for $\dot{X}_R$ and $\dot{Y}_R$ can be written symbolically as:

$$\left.\begin{aligned} \dot{X}_R &= g_1(X, Y, Z, \dot{Z}, t) \\ \dot{Y}_R &= g_2(X, Y, Z, \dot{Z}, t) \end{aligned}\right\} \qquad (3\text{-}3)$$

An important point concerning notation: In Eq. (3-2) the variables have the subscript i , indicating actual injection values, i.e., at actual injection the two miss components expressed as functions of the seven variables actually vanish. In equation (3-3) however, the 5 variables on the right do not have any subscript. This is to indicate that any set of these 5 variables would lead to a lunar hit if $\dot{X}$ has the value $\dot{X}_R$ and $\dot{Y}$ the value $\dot{Y}_R$.

The preceding discussion provides the background necessary for understanding the basis of the guidance philosophy. The general idea is illustrated in figure 3-2. The navigation portion of the system continuously supplies outputs of all seven navigational variables X , Y , Z , $\dot{X}$, $\dot{Y}$, $\dot{Z}$, t . The five variables we have designated as independent variables (X , Y , Z , $\dot{Z}$, t) are used as the inputs to the computation of $\dot{X}_R$ and $\dot{Y}_R$. The actual manner of computing $\dot{X}_R$ and $\dot{Y}_R$ used in the airborne guidance computer will be considered later in this volume. For the present, it suffices to say that appropriate equations exist for this purpose, which for the present, we indicate schematically by equation (3-3). The

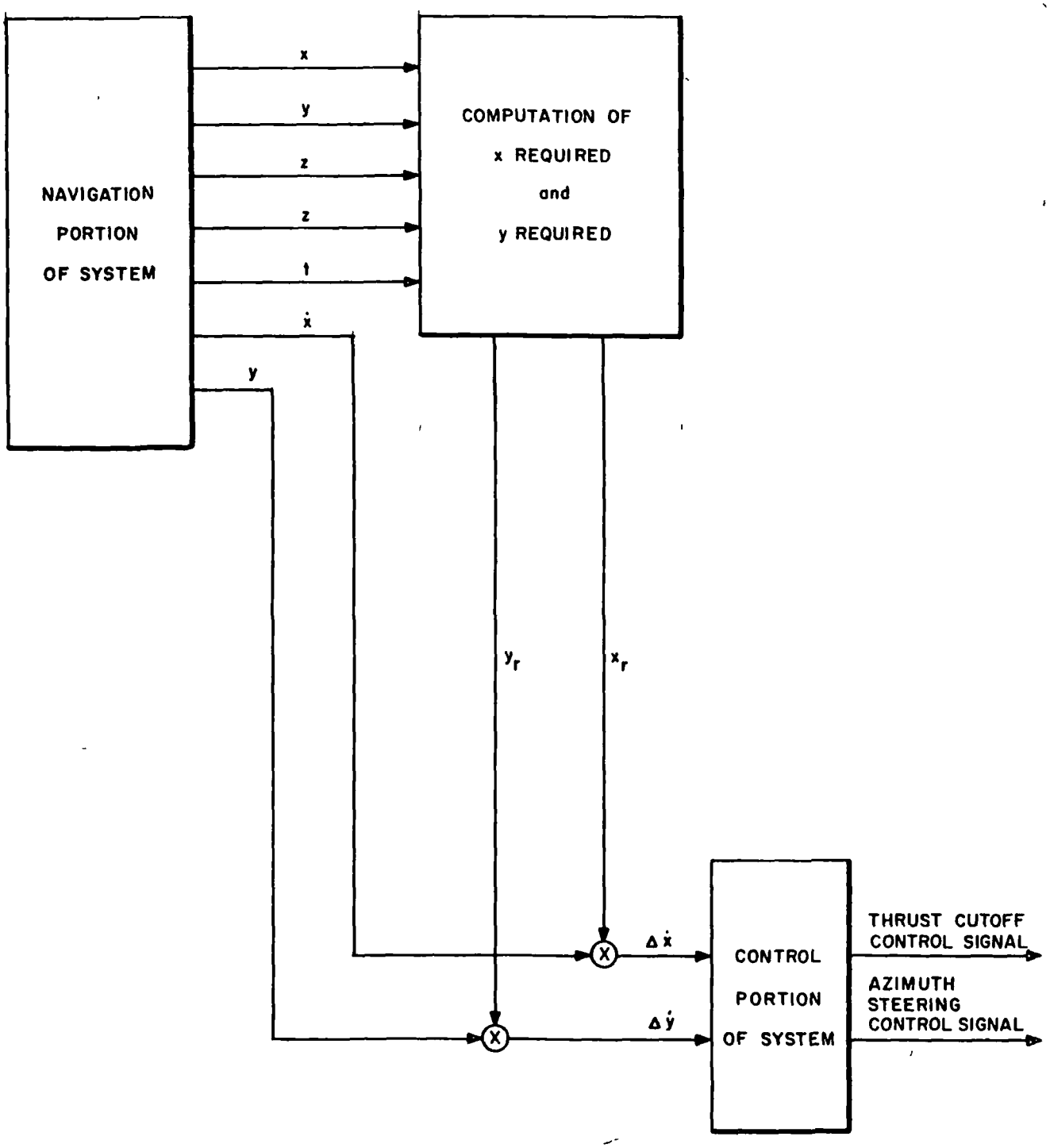

Figure 3-2 Simplified Block Diagram of Guidance Computations (Modification of $\Delta\dot{y}$ to Eliminate Spurious Azimuth Steering Not Shown)

computed values of $\dot{X}_R$ and $\dot{Y}_R$ are compared respectively with the actual values of $\dot{X}$ and $\dot{Y}$ derived from the navigation portion of the system. The resulting differences $\Delta\dot{X}$ and $\Delta\dot{Y}$ are inputs to the control portion of the system. The quantities $\Delta\dot{X}$ and $\Delta\dot{Y}$ are known as the X and Y velocity-to-be-gained, respectively.

As explained previously, the two missile control parameters employed to insure that the conditions existing at injection will result in a lunar impact are thrust cutoff and azimuth steering. Within the control portion of the system, the $\Delta\dot{X}$ and $\Delta\dot{Y}$ information are used to generate azimuth steering and thrust cutoff signals.

Because of the key role played by the quantities $\Delta\dot{X}$ and $\Delta\dot{Y}$, this technique of deriving guidance signals is known as the velocity-to-be-gained approach.

(Note: Another approach to the derivation of guidance signal was considered briefly and rejected early in the study. This technique is known as the geometrical miss method and proceeds as follows: At each point in the powered flight, the seven injection variables are considered as a tentative set of injection conditions. The seven variables are used to compute, via equations (2-1) (2-2), the two components of the miss ϵ_V and ϵ_H, which would result if thrust were terminated at that instant. The resulting ϵ_V and ϵ_H signals are fed to the control portion of the system where they are used to control the azimuth steering and thrust cut-off so that at injection both ϵ_V and ϵ_H are zero. The main reason for rejecting this method is the following: It will be recalled that the miss components are evaluated at the predicted moment when the vehicle would pierce a sphere whose center is at the center of the Earth, and whose radius equals the distance of the moon from the center of the Earth. During the early portion of the powered flight, the missile does not have enough velocity to reach this radius if thrust were terminated. In fact, on a well designed trajectory, thrust termination should occur only several seconds after this velocity has been reached. Thus, until a few seconds before cut-off the miss components and therefore, guidance data, cannot be computed. Of course, one might considering salvaging the geometrical miss method by

defining the miss components in a different way so that the miss can be computed throughout the powered flight. As explained previously, the particular choice which was made for the definition of the miss components ϵ_V and ϵ_H leads to considerable simplification of the mathematics. Hence, the geometrical miss method was abandoned and effort was concentrated solely on the velocity-to-be-gained approach explained above.

3.5 Use of Velocity-to-be-Gained Data to Control Missile Flight

In the preceding section, the method of processing the navigational data to arrive at the guidance signals $\Delta\dot{X}$ and $\Delta\dot{Y}$ has been explained. We proceed now to consider the manner in which these guidance signals are used to control the azimuth steering and thrust termination so that proper injection conditions result. Before proceeding directly to this matter, it is necessary to consider a problem which has not been previously discussed.

It will be recalled that, of the seven variables, two must be singled out as dependent variables if lunar interception is to be accomplished. Let us explain our choice of $\dot{X}$ and $\dot{Y}$ for this role of dependent variables. This choice is closely related to the use of thrust cut-off time and azimuth steering as the missile control parameters for insuring lunar interception. It is clearly desirable that there be as little coupling as possible between the thrust cut-off control system and the azimuth steering control system. Choosing $\dot{X}$ and $\dot{Y}$ as dependent variables results in a straightforward, effective decoupling of these two control systems. This can be seen as follows: The following table shows the effect on $\dot{X}$ and $\dot{Y}$ of incremental changes in burning time and azimuth heading of the velocity vector. These data were evaluated at the injection point of the nominal trajectory.

	effect on $\dot{X}$ (ft/sec.)	effect on $\dot{Y}$ (ft/sec.)
1 sec. burning time	1350	0
1 deg. azimuth change of velocity vector	5.3	615

It can be seen that $\dot{X}$ is very strongly affected by burning time while $\dot{Y}$ is very strongly affected by azimuth heading. Conversely, $\dot{Y}$ is not affected by burning time, nor is $\dot{X}$ appreciably affected by azimuth heading. Hence, the following simple rules for employing $\Delta\dot{X}$ and $\Delta\dot{Y}$ to control engine cut-off time and azimuth steering emerge·

.... $\Delta\dot{Y}$ is used to control azimuth steering.

.... $\Delta\dot{X}$ is used to control engine cut-off time

The abovementioned decoupling of the azimuth and engine cut-off controls can now be understood:

....A non-zero $\Delta\dot{Y}$ is used to steer the missile in azimuth. This steering (see above table) tends to drive $\Delta\dot{Y}$ toward zero, but has virtually no effect on $\Delta\dot{X}$.

....A non-zero $\Delta\dot{X}$ indicates that the missile engines should be allowed to continue burning. The added velocity tends to drive $\Delta\dot{X}$ toward zero, but has no effect on $\Delta\dot{Y}$.

Let us examine more closely just how the $\Delta\dot{Y}$ signal is used to control azimuth steering. A naive approach would be to attempt to steer the missile in azimuth in a manner such that $\Delta\dot{Y}$ is held continually at zero. We then simply let burning continue until $\Delta\dot{X}$ is zero, and then cut-off the engines. Practically, this technique of azimuth steering has severe disadvantages, since it demands excessive steering in azimuth with consequent waste of fuel and reduction in payload capability. A much more effective scheme is based on the following considerations· Actually, it is not necessary to maintain $\Delta\dot{Y}$ at zero throughout the powered flight. All that is required is that $\Delta\dot{Y}$ be zero at the moment of cut-off, i.e., when $\Delta\dot{X}$ is also zero. Therefore, as long as $\Delta\dot{Y}$ is proceeding toward zero satisfactorily so that it will be zero when $\Delta\dot{X}$ is zero, no change in azimuth heading is called for. How do we attach precise significance to the phrase "proceeding toward zero satisfactorily"?

In order to answer this question, it is necessary to introduce the concept of the nominal trajectory. Imagine a missile whose performance is in precise conformance with design concerning acceleration time history, pre-programmed pitch profile, and other characteristics which affect its motion in space as a function of time. For such a missile, one can design a trajectory by properly choosing the pitch profile, constant azimuth heading, launch time, and cut-off time so that the vehicle would impact the moon without the sophisticated guidance system we are considering. This trajectory is known as the nominal trajectory. In a certain sense, the sophisticated guidance system is necessary only because of uncontrollable departures from the nominal trajectory such as delayed launch time, errors in pitch program, deviations in engine performance, etc. The nominal trajectory is an important concept which will be considered more fully later in this volume.

The nominal trajectory is used to establish rules for determining whether $\Delta\dot{Y}$ is "proceeding toward zero satisfactorily". Using digital simulation techniques (see Volume V), the flight of a missile along the nominal trajectory is simulated. The guidance signals $\Delta\dot{X}$ and $\Delta\dot{Y}$ are computed as the missile "flys" the nominal trajectory, but are not allowed to control the flight. Given the values of $\Delta\dot{X}$ and $\Delta\dot{Y}$ corresponding to each point on the nominal trajectory we can plot a curve of $\Delta\dot{Y}$ as a function of $\Delta\dot{X}$. Symbolically:

$$\Delta\dot{Y}_{NOM} = f(\Delta\dot{X}_{NOM}) \qquad (3\text{-}4)$$

The subscript " NOM " indicates that the functional relation (3-4) is strictly true only for the nominal trajectory. It is assumed that an actual off-nominal trajectory is similar enough to the nominal trajectory so that if $\Delta\dot{Y}$ is "proceeding toward zero satisfactorily" on an off-nominal trajectory $\Delta\dot{Y}$ will be functionally related to $\Delta\dot{X}$ by the relation (3-4) even though the function f was derived for the nominal trajectory. Therefore,

excessive azimuth steering is eliminated by replacing the azimuth steering signal $\Delta\dot{Y}$ by the modified signal $\Delta\dot{Y} - f(\Delta\dot{x})$. It is important to note that the approximation involved in deriving the function $f(\Delta\dot{X})$ only from the nominal trajectory does not affect the accuracy of the guidance data at the all-important injection point (since $f(\Delta\dot{X})$ is zero at that point), but simply means that a small amount of spurious azimuth steering remains.

One final point: It is assumed that the relation (3-4) can be approximated by a straight line, i.e., the azimuth steering signal is taken to be $\Delta\dot{Y} - K\Delta\dot{X}$, where K is an appropriately chosen constant. The azimuth angle, β, of the missile body axis is made proportional to the azimuth steering signal, $\Delta\dot{Y} - K\Delta\dot{X}$, via the relation

$$\beta = K_{\alpha_1}(\Delta\dot{Y} - K\Delta\dot{X}) \qquad (3\text{-}5)$$

(Note: In the Arma system the missile azimuth angle β and the elevation angle α are measured relative to the inertial platform coordinate system as shown in figure 3-3. β is measured counterclockwise from the X axis and α is positive upwards.)

The abovementioned straight line fit to $f(\Delta\dot{X})$ is good only in the later portion of the powered flight. Therefore, until this time is reached, the missile is steered in azimuth so as to keep it in the X-Z plane. An evaluation of the performance of this azimuth steering scheme is presented later in this volume.

3.6 Computation of Guidance Data by the Airborne Computer

The feasibility of the guidance concept outlined above and illustrated schematically in figure 3-3 clearly is critically dependent on the accuracy with which the required X and Y velocity components ($\dot{X}_R$ and $\dot{Y}_R$) can be computed by the airborne guidance computer. Up to this point, the dependence of these two quantities on the five independent navigation variables X, Y, Z, $\dot{Z}$, t has only been indicated schematically as

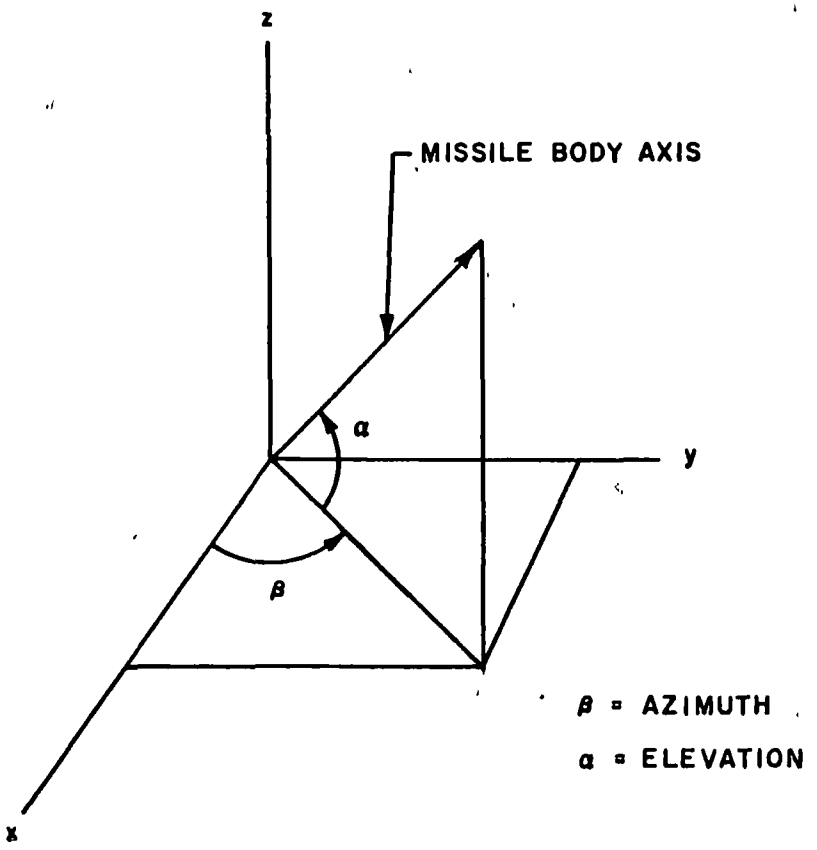

Figure 3-3 Definition of Missile Body Axis Azimuth and Elevation Angles Relative to Inertial Platform Coordinate System

$$\left.\begin{aligned} \dot{x}_r &= g_1(x, y, z, \dot{z}, t) \\ \dot{y}_r &= g_2(x, y, z, \dot{z}, t) \end{aligned}\right\} \qquad (3\text{-}6)$$

In this section, we shall describe the technique employed in the airborne guidance computer for computing these two quantities.

The method employed is the so-called polynomial expansion method which has gained wide acceptance in certain military missile applications. It was considered to be of great interest to investigate the applicability of the polynomial expansion method to injection guidance for space missions, in particular the lunar interception mission of the present study.

The polynomial expansion method is an approximation technique and depends heavily upon the assumption that an actual off-nominal trajectory flown by a real missile will not differ too markedly from the idealized nominal trajectory introduced earlier in this volume.

The injection conditions of the nominal trajectory are designated x_n, y_n, z_n, $\dot{x}_n$, $\dot{y}_n$, $\dot{z}_n$, t_n. At any point on a real off-nominal trajectory, the five independent navigation variables are x, y, z, $\dot{z}$, t. It is assumed that the functional dependence of $\dot{x}_r$ and $\dot{y}_r$ on these five variables can be expressed in terms of polynomial expansions in which the variables are the five differences

$$\left.\begin{aligned} \Delta x &= x - x_n \\ \Delta y &= y - y_n \\ \Delta z &= z - z_n \\ \Delta \dot{z} &= \dot{z} - \dot{z}_n \\ \Delta t &= t - t_n \end{aligned}\right\} \qquad (3\text{-}7)$$

Specifically, it is assumed that $\dot{x}_r$ and $\dot{y}_r$ can be expressed in the form:

$$\left.\begin{aligned} \dot{x}_r &= K + \sum_{l,m,n,p,q} K_{l,m,n,p,q}\, \Delta x^l \Delta y^m \Delta z^n \Delta \dot{z}^p \Delta t^q \\ \dot{y}_r &= C + \sum_{n,s,u,v,w} C_{n,s,u,v,w}\, \Delta x^n \Delta y^s \Delta z^u \Delta \dot{z}^v \Delta t^w \end{aligned}\right\} \quad (3\text{-}8)$$

In order to make the notation of equation (3-8) clear, we illustrate some typical terms.

....term in $\dot{x}_r$ which is linear in $\Delta y = K_{0,1,0,0,0}\, \Delta y$

....term in $\dot{x}_r$ which is quadratic in $\Delta \dot{z} = K_{0,0,0,2,0}\, \Delta \dot{z}^2$

....term in $\dot{y}_r$ which is quadratic in Δx and linear in Δy

$= C_{2,1,0,0,0}\, \Delta x^2 \Delta y$

Of course, many of the K's and C's in equation (3-8) are zero since the corresponding terms are not present in the polynomial expansions.

In considering a practical application of evaluating $\dot{x}_r$ and $\dot{y}_r$ by means of the polynomial expansion technique, we find there are two all-important steps in deriving the guidance expansions:

....The decision as to the form of the expansions, i.e., which terms shall be present in the expansions.

....The derivation of the numerical values of the constant K's and C's.

The form of the expansions is to a great extent a matter of engineering judgment. While a small number of terms is desirable in order to reduce the burden placed on the airborne

computer, it is clear, on the other hand, that the larger the number of terms, the more accurately the approximate expansions can be made to fit the exact analytic mathematical solution for $\dot{x}_r$ and $\dot{y}_r$. The effect of adding and deleting specific terms in the expansions will be discussed with quantitative results later in this volume.

Concerning the determination of the numerical values of the constants in the expansions, two techniques were investigated:

....The first technique is the Taylor series method (which was studied and rejected early in the study). In this technique, the series expansions (3-8) are considered to be truncated Taylor series expansions about the nominal injection point. Thus, the constants without subscripts are interpreted as the values of $\dot{x}_r$ and $\dot{y}_r$ at the nominal injection point:

$$K = \dot{x}_n \, , \quad C = \dot{y}_n$$

The constants with subscripts have the significance of derivatives of $\dot{x}_r$ and $\dot{y}_r$ with respect to the appropriate independent variables. Thus, for example, the term in $\dot{x}_r$ which is linear in Δy is written

$$\left(\frac{\partial \dot{x}_r}{\partial y} \right)_n \Delta y$$

where the subscript n indicates that the partial derivative is evaluated at the nominal injection point. The partial derivatives are evaluated by numerical differentiation.

The Taylor series method of fixing the constants places great emphasis on goodness of fit in a small region around the nominal injection conditions, since the partial derivatives are evaluated at the nominal injection point. This leads to extremely accurate guidance computations close to the nominal trajectory, but the fit goes bad rather rapidly as off-nominal deviations in missile behavior are increased. It was considered highly desirable to derive the constants using a method which sacrifices some of the goodness of fit close to nominal conditions in order to gain a better fit at large off-nominal conditions. This consideration led to the abandonment of the Taylor series method and the development of the second method.

. . .The second method of deriving the constants in the expansions is the least squares method. In this technique, we accept the fact that the polynomial expansions for $\dot{X}_r$ and $\dot{Y}_r$ are approximations and will result in errors. Thus, we consider a particular set of off nominal deviations ΔX_i, ΔY_i, ΔZ_i, $\Delta \dot{Z}_i$, Δt_i. Let the analytically correct value of $\dot{X}_r$ for this point be designated $\dot{X}_{ri}$. Then the polynomial expansion for $\dot{X}_r$ will not yield $\dot{X}_{ri}$ precisely but will have a certain error ϵ_i. That is

$$\dot{X}_{ri} - \left(K + \sum_{l,m,n,p,q} K_{l,m,n,p,q} \Delta X_i^l \Delta Y_i^m \Delta Z_i^n \Delta \dot{Z}_i^p \Delta t_i^q \right) = \epsilon_i \quad (3\text{-}9)$$

The constants K and $K_{l,m,n,p,q}$ are determined as follows: We select a large number of sets of deviations ΔX_i, ΔY_i, ΔZ_i, $\Delta \dot{Z}_i$, Δt_i from the nominal injection conditions X_n, Y_n, Z_n, $\dot{Z}_n$, t_n. It is postulated that the constants shall be chosen so that the summation for all these sets of deviations

$$\sum_i \epsilon_i^2$$

is minimized. This is the technique for determining the constants which is recommended as a result of this study, assuming that guidance is accomplished by equations of the polynomial expansion type. The details of the mathematics and the IBM 704 programs used in deriving the constants are presented in Volume V. The selection of the off-nominal deviations used in the least squares analysis is an extremely important matter which will be discussed in greater detail later in this volume.

In order to carry out the least squares analysis, it is first necessary to compute correct values of $\dot{X}_r$ and $\dot{Y}_r$ corresponding to the sets of off nominal conditions. The method used in this study for computing these values on

the IBM 704 computer is illustrated schematically in Figure 3-4. It will be recalled that the two components of the predicted vehicle miss at the moon, ϵ_V and ϵ_H, are uniquely determined functions of all seven navigation variables. The computation of $\dot{X}_r$ and $\dot{Y}_r$ is started by using the five independent variables and assumed values of $\dot{X}_r$ and $\dot{Y}_r$ to determine the miss components ϵ_V and ϵ_H. The miss components are then used to generate corrections to the assumed values of $\dot{X}_r$ and $\dot{Y}_r$. The iterative cycle is started over again using the five independent variables and the corrected values of $\dot{X}_r$ and $\dot{Y}_r$. The iterative procedure is repeated until the corrected values of $\dot{X}_r$ and $\dot{Y}_r$ lead to values of ϵ_V and ϵ_H which are sufficiently small. When this occurs, the last corrected values are taken as the values of $\dot{X}_r$ and $\dot{Y}_r$ corresponding to the five independent variables. The details of this calculation are given in Volume V, Section 3.

Conceivably, the above iterative method might be used as the basis, in an airborne computer, of computing the $\dot{X}_r$ and $\dot{Y}_r$ required for the guidance scheme in Figure 3-2. In fact, Arma has under development a flexible airborne computer which, on the basis of preliminary studies, will be capable of solving the analytic equations for the miss and performing the indicated iterative process to arrive at $\dot{X}_r$ and $\dot{Y}_r$.

3.7 Operational Concept for Delayed Launch Missions

It is clear that the guidance system concept under consideration is heavily dependent on the idea of a nominal trajectory. Obviously, one of the parameters defining the nominal trajectory is the nominal launch time. When we consider actual missiles flying actual off-nominal trajectories, it is necessary to consider the possibility of a deviation from nominal launch time, such as might be caused by a delay in count-down. In this section, we shall consider a technique which allows the guidance system to make allowances for such delays in launching.

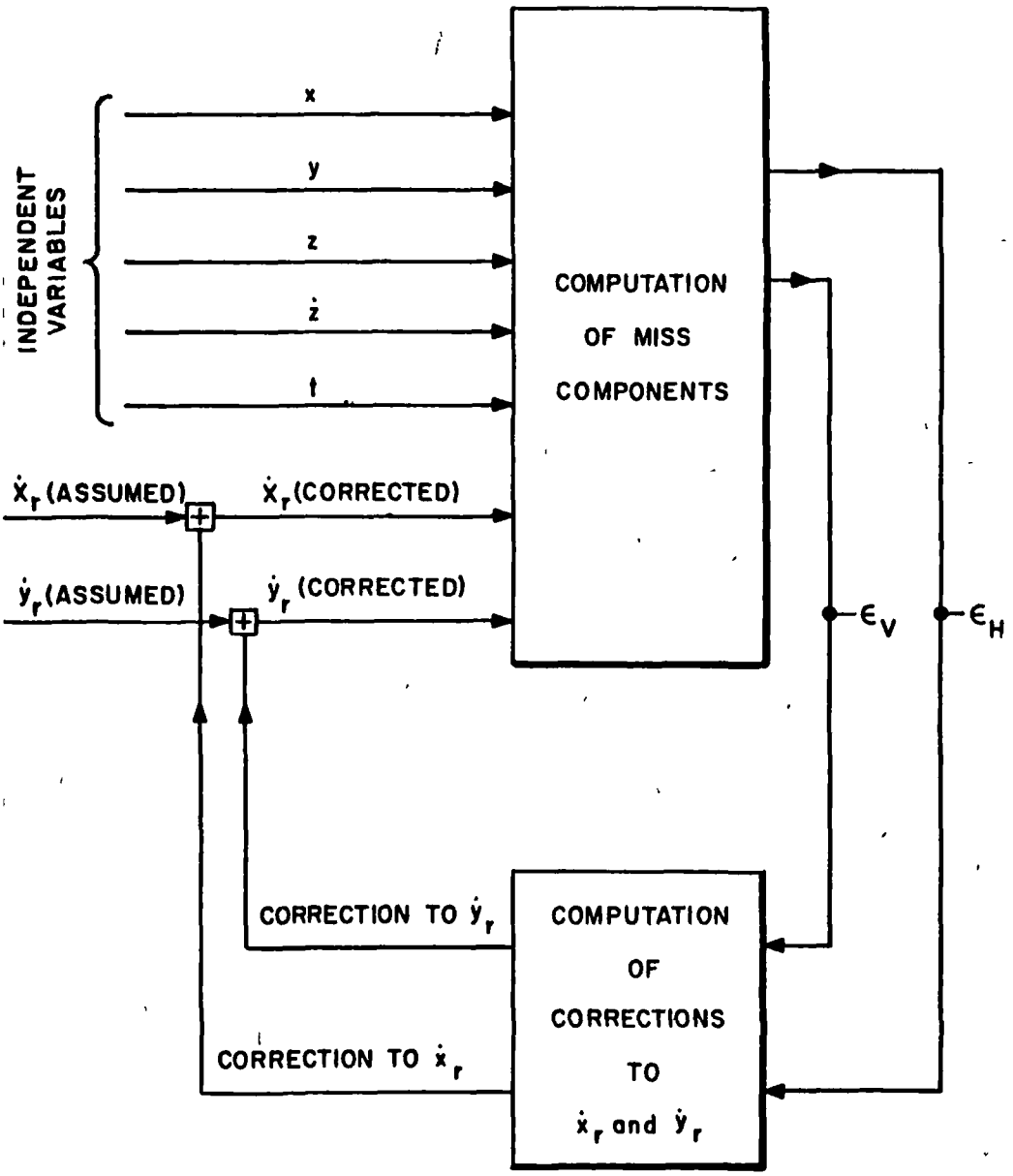

Figure 3-4 Iterative Method for Computing $\dot{x}_r$ and $\dot{y}_r$

On a nominal mission, several key events occur at the nominal launch time. Those events pertinent to the present discussion are:

....The platform is cast free in inertial space, defining the coordinate system in which position and velocity are measured.

....The navigation portion of the system starts computing the position and velocity of the vehicle relative to the system's inertial coordinate system.

....The clock which establishes the system time reference is started.

....The missile pitch attitude program is started.

A very simple procedure which might be conceived for dealing with a late launch is the following: When nominal launch time is reached, the guidance system is placed in full operation even though the missile does not leave the launch pad. The navigation system then keeps track of position and velocity in the inertial coordinate system starting at nominal launch time and simply continues until lift-off and on into the powered flight. Thus, the only effect of the late launch is that the trajectory has off-nominal position and velocity versus time as seen in the inertial coordinate system. As long as these off nominal position and velocity deviations are not too large, i.e., as long as the delay in launch time is not too great, the errors of the polynomial expansion approximations for $\ddot{X}_r$ and $\ddot{Y}_r$ remain within acceptable limits. If, however, the launch delay is too great (10 minutes is operationally quite possible), the guidance equations become inaccurate. Thus, the above simple technique is not completely adequate, since from an operational point of view approximately 20 minutes leeway in launching time (±10 minutes from optimum launch time) is highly desirable. Therefore, the following additions to the above simple method are necessary:

The general idea can be explained with the aid of Figure 3-5. First, the earliest and latest possible launch times are determined, based on operational limitations. These are

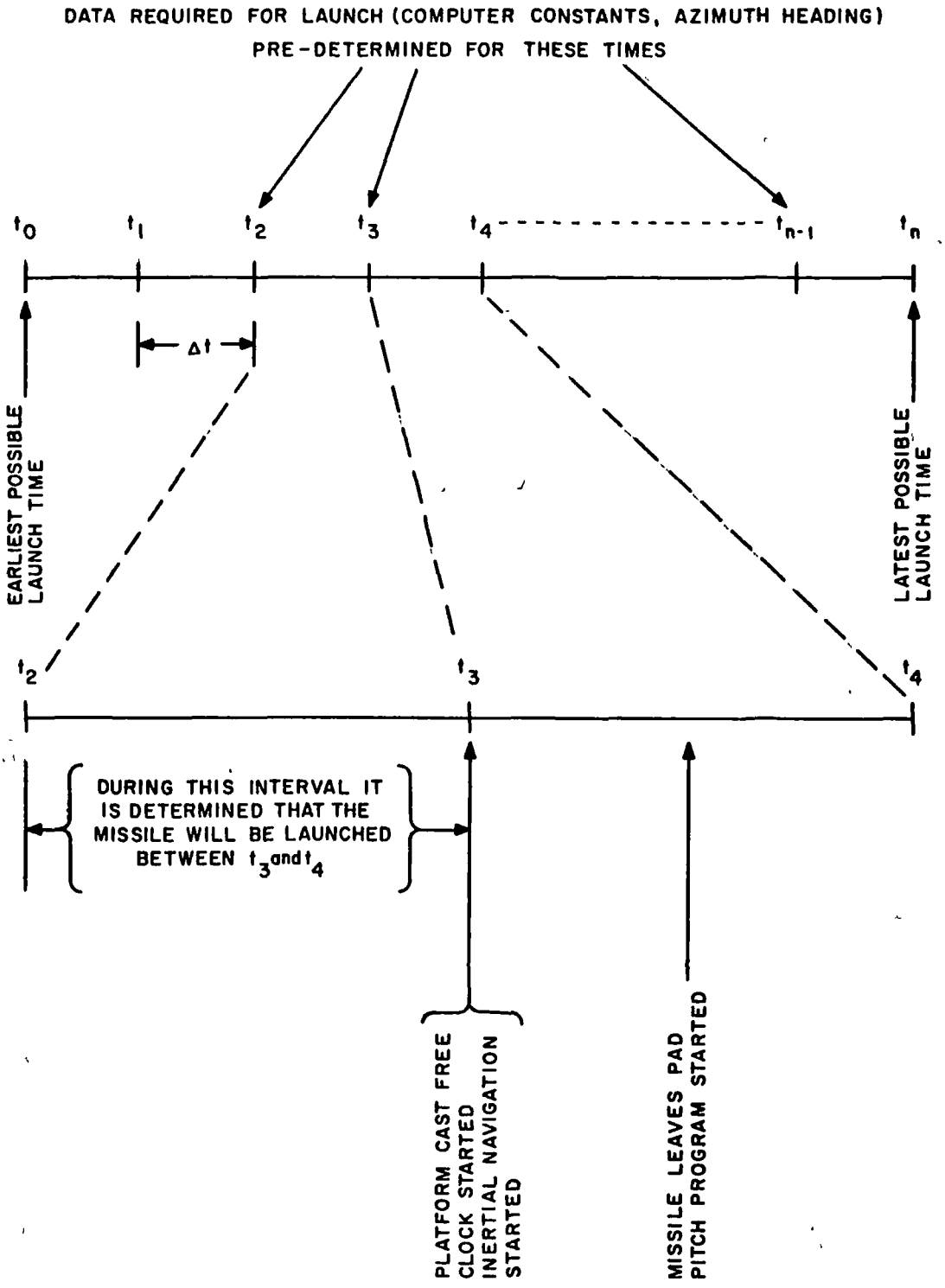

Figure 3-5 Operational Concept for Delayed Launch Time

designated t_o and t_n respectively. (The times t_o and t_n will be determined primarily by the fact that propulsion requirements increase as launch time deviation from optimum increases Typically, t_o and t_n will be separated by 20 minutes.) Next, using the method described at the beginning of this section. i.e., placing the guidance system into full operation at a nominal launch time, even though the missile is still on the ground, we determine how large a delay in launch can be tolerated before the polynomial guidance expansions become excessively inaccurate. This time delay is designated Δt . (Data presented later definitely establishes Δt as longer than 0.5 minutes, but the upper limit has not been determined during the present study.) Next, the interval between t_o and t_n is divided into subintervals of size Δt . The dividing times are designated t_1 , t_2 , etc. A nominal trajectory is derived for <u>each</u> of the times t_o , t_1 , t_2 , ... t_n . For each of these nominal trajectories, the launch data required for the guidance system (constant coefficients for the polynomial expansions, correct azimuth orientation of the platform and initial velocity components in inertial coordinates) are determined. Thus, the missile can be launched at any of the times t_o , t_1 , t_2 , ... t_n with no error due to delayed launch since each of these times is a new nominal launch time. If we launch <u>between</u> two of the nominal times, the errors will be acceptable since we are always within a time Δt of the last nominal launch time.

Let us trace through the operation for a typical launch: First of all, the platform is aligned to the vertical. Prior to the time t_o, the azimuth orientation of the platform, the initial velocity values and the polynomial expansion constants in the computer are those appropriate to the time t_o . When time t_o is reached, the polynomial constants and initial velocity data stored in the computer are switched to those appropriate to t_1 , and the platform is rotated in azimuth so that it will be properly aligned for a launch at time t_1 . This procedure continues, i.e., as the beginning of each interval is passed, the computer constants, initial velocity, and azimuth alignment are switched to those appropriate to the beginning of the <u>next</u> interval. Suppose, for example (see Figure 3-5), that the count down procedure has reached a point so that

during the interval t_2 to t_3 it is definite that the missile will be launched between t_3 and t_4 . At t_3, the initial velocity components, the constants in the computer, and the azimuth alignment are correct for that instant. As t_3 is passed, the platform is cast free in inertial space, the system's time reference clock is started, and inertial navigation in inertial coordinates is started. Finally, at some time between t_3 and t_4, the missile lifts off and the missile pitch attitude program is started. The fact that the computer constants must be changed implies a degree of flexibility in the computer. The type of memory medium which is recommended to satisfy this requirement is discussed in Volume IV of this report.

3.8 Summary of Guidance System Concept

We close this discussion of the technical approach to the guidance system formulation with a summary of the main features of the system. These are shown schematically in Figure 3-6.

Prior to launch, the platform is aligned to the vertical. When the earliest possible launch time is reached (t_0 in Figure 3-5), the ground support equipment aligns the platform in azimuth, sets initial velocity data in inertial space into the navigation portion of the computer, and sets in the constants of the polynomial expansions, all of this information being appropriate for launch at t_1 . When t_1 is passed, the system is re-set for a launch at t_2 . Finally, definite word is reached from the appropriate count-down personnel that launch will occur during the next interval Δt . When the time corresponding to the beginning of this interval is reached, a signal is sent to the guidance system from the ground support equipment. This signal casts the platform free in inertial space, freezes the initial inertial velocity data, starts the computer time reference, and starts navigating the missile in inertial space. Finally, at the moment of lift-off, the ground support equipment sends a signal to the guidance system which starts the pitch programmer, and the missile is on its way under control of the inertial guidance system.

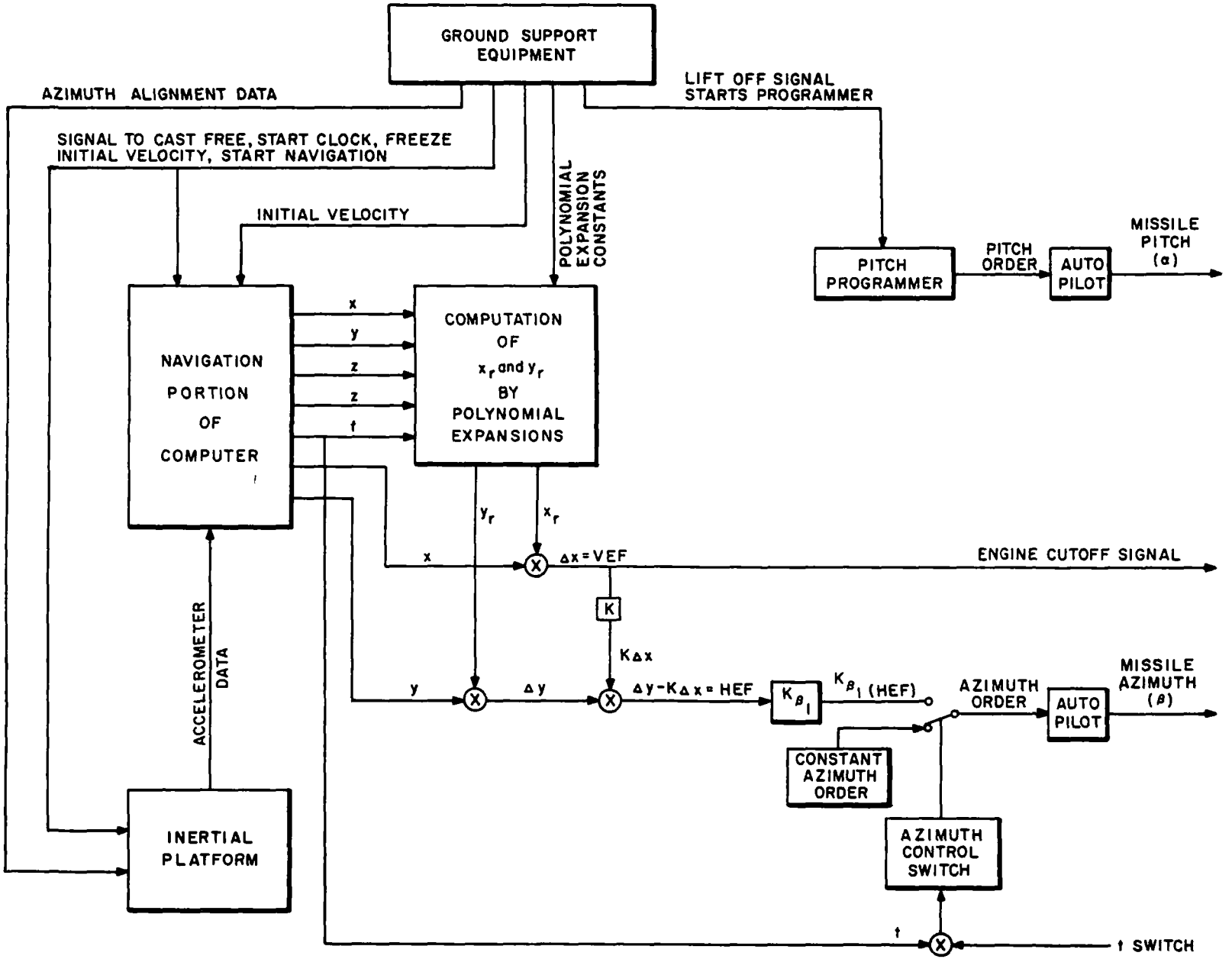

Figure 3-6 Block Diagram of Guidance System

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Throughout the powered flight, the missile pitch attitude is controlled by the pitch programmer via the missile autopilot. Actually, most of the auto-pilot functions will be performed by the inertial platform, the only additional components required being body mounted rate gyros for damping.

The navigation portion of the system throughout the powered flight delivers continuous data on the seven navigation variables X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$, t. The five independent variables X, $\dot{Y}$, Z, $\dot{Z}$, t, are used as inputs to the computation of $\dot{X}_r$ and $\dot{Y}_r$, the values of $\dot{X}$ and $\dot{Y}$ which are required for a lunar impact assuming the five independent variables are given. The computation of $\dot{X}_r$ and $\dot{Y}_r$ as functions of the five independent variables is carried out by the polynomial expansion technique described earlier. $\dot{X}_r$ is compared with the present value of $\dot{X}$ coming from the navigation portion of the system. The difference $(\Delta\dot{X})$, known as the VEF (vertical error function), causes the engine thrust to be cut-off when VEF equals zero. Probably, in a practical application, the missile would carry vernier engines. In such a case, the main engines would be shut down when VEF reaches a certain pre-determined value, and thrust would be stopped completely when VEF equals zero. In the present study, the effect of vernier engines has not been included, since they do not affect the guidance concept in any essential characteristic.

As described earlier, the azimuth steering scheme which is used to null $\Delta\dot{Y}$ and also avoid spurious azimuth steering (see Section 3.5) employs certain mathematical approximations which are not sufficiently accurate during the early part of the flight. Therefore, after lift-off, azimuth control is accomplished through some simple technique, such as maintaining a constant missile azimuth heading. When switching time is reached (t_{sw} in Figure 3-6), azimuth control is switched to the method used for nulling $\Delta\dot{Y}$ at injection.

In this method, the Y velocity to be gained, $\Delta\dot{Y}$, is modified in a manner which prevents excessive azimuth steering. The modified signal, $\Delta\dot{Y} - K\Delta\dot{X}$, is known as the HEF, or horizontal error function. The azimuth angle order is derived by multiplying HEF by a scaling constant $K\beta_1$.

SECTION 4

EVALUATION OF GUIDANCE SYSTEM PERFORMANCE

4.1 General Remarks

Up to this point in the present volume, we have considered the basic philosophy and the general technical approach to the formulation of an injection guidance system for the lunar impact mission. We have explained how missile pitch, azimuth, and thrust cut-off are derived, how the guidance computer performs its functions, and the operational concept for compensating for deviations from nominal launch time.

It is necessary now, in order to arrive at a realistic evaluation of the guidance system concept, to leave these general considerations of technical philosophy and proceed to a detailed examination of the expected quantitative performance of the system.

The primary tool in performing this evaluation is the simulation of the missile as it would behave if controlled by the guidance system of the present study. This simulation effort was carried out with the aid of the IBM 704 computer.

It was explained earlier in this volume that the guidance concept places heavy emphasis on the idea of a nominal trajectory. Obviously, if a missile actually flew a precise nominal trajectory, i.e., if engine performance, pitch program, launch time, etc., were precisely controlled, then even an extremely crude guidance system concept will result in accurate performance. Therefore, the acid test of the guidance concept is its performance when confronted with off-nominal deviations in mass flow rate, fuel specific impulse, launch time, etc. Hence, as will be seen later, great emphasis in the simulated flights is placed on these off-nominal effects.

In performing the study, the simulated flights were in two basic groups:

....coplanar flights - In this group, several simplifying assumptions were made. It was assumed that the moon's orbital plane coincides with the Earth's equatorial plane and that the launch site is on the equator. These assumptions allow the thrust cut-off guidance to be evaluated separately, since azimuth steering is trivial in this case. Also, the simplifications in the analysis allowed the coplanar cases to be investigated rapidly and to serve as a guide in setting up the non-coplanar investigation.

....non-coplanar flights - In this group, the realistic situation was simulated. That is, the moon's orbital plane was tipped at an appropriate angle relative to the Earth's equatorial plane, and the launch site latitude was taken close to that of Cape Canaveral. In carrying out this work, the complete three dimensional geometrical relationships were taken into account. The non-coplanar cases were necessary for a complete evaluation including the azimuth steering scheme.

The results in this volume are concerned with the expected performance of the guidance system concept assuming perfect components (gyros, accelerometers, etc.). Clearly, a detailed accuracy analysis of component errors is also of prime importance. A discussion of the guidance system equipment and its accuracy characteristics is given in Volume IV of this report.

4.2 Outline of System Evaluation Technique

Because of the key part played by simulation and computation in this study, it is possible to trace out the entire formulation and evaluation of the guidance concept in terms of the computation programs. This will be helpful in following the discussion in the remainder of this volume. In Figure 4-1 is shown the flow of information between the various programs.

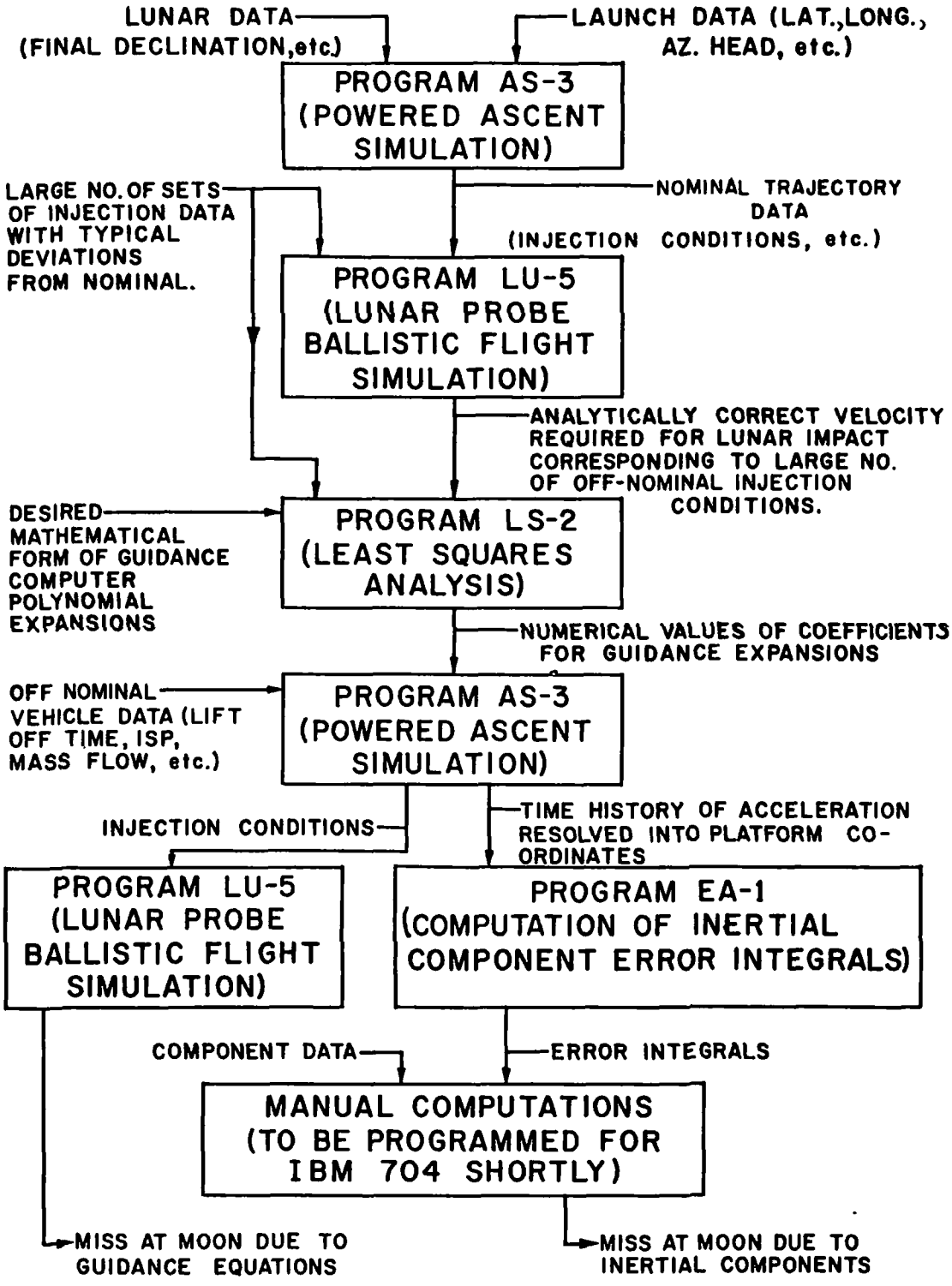
LUNAR DATA (FINAL DECLINATION,etc.)
LAUNCH DATA (LAT.,LONG., AZ. HEAD, etc.)
PROGRAM AS-3 (POWERED ASCENT SIMULATION)
LARGE NO. OF SETS OF INJECTION DATA WITH TYPICAL DEVIATIONS FROM NOMINAL.
NOMINAL TRAJECTORY DATA (INJECTION CONDITIONS, etc.)
PROGRAM LU-5 (LUNAR PROBE BALLISTIC FLIGHT SIMULATION)
ANALYTICALLY CORRECT VELOCITY REQUIRED FOR LUNAR IMPACT CORRESPONDING TO LARGE NO. OF OFF-NOMINAL INJECTION CONDITIONS.
DESIRED MATHEMATICAL FORM OF GUIDANCE COMPUTER POLYNOMIAL EXPANSIONS
PROGRAM LS-2 (LEAST SQUARES ANALYSIS)
NUMERICAL VALUES OF COEFFICIENTS FOR GUIDANCE EXPANSIONS
OFF NOMINAL VEHICLE DATA (LIFT OFF TIME, ISP, MASS FLOW, etc.)
PROGRAM AS-3 (POWERED ASCENT SIMULATION)
INJECTION CONDITIONS
TIME HISTORY OF ACCELERATION RESOLVED INTO PLATFORM CO-ORDINATES
PROGRAM LU-5 (LUNAR PROBE BALLISTIC FLIGHT SIMULATION)
PROGRAM EA-1 (COMPUTATION OF INERTIAL COMPONENT ERROR INTEGRALS)
COMPONENT DATA
ERROR INTEGRALS
MANUAL COMPUTATIONS (TO BE PROGRAMMED FOR IBM 704 SHORTLY)
MISS AT MOON DUE TO GUIDANCE EQUATIONS
MISS AT MOON DUE TO INERTIAL COMPONENTS

FIG. 4-1 INFORMATION FLOW THROUGH SIMULATION AND COMPUTATION PROCEDURES

The first step is the use of the Powered Ascent Simulation Program (AS-3) in developing a nominal trajectory. The following information is required to start this process

. . . . the desired impact time and impact declination (usually taken as maximum negative lunar declination during the month in question).

. . . the latitude and longitude of the launch site.

The vehicle is "flown" in a zero lift turn with constant inertial azimuth heading, and thrust is terminated when the subsequent ballistic flight will cause the vehicle to reach the radius of the moon's orbit when it passes through the desired impact declination. If some particular characteristic of the nominal flight is desired (e.g., a specified ballistic flight time, powered flight time, etc.), the nominal powered trajectory satisfying the desired condition is found through successive approximations by varying the azimuth and the no-lift turn until the desired trajectory is found. For feasibility studies such as the present one, we assume that the moon follows a "pseudo-ephemeris", i.e., that the moon will be at the desired declination at the time the nominal missile reaches that declination. The AS-3 program, however, can be used to make further modifications to the trajectory in order to be consistent with the actual lunar ephemeris.

As indicated earlier, the guidance equations are approximate relations involving numerical coefficients. The next step in the formulation of the system concept is the determination of these numerical constants. For the least squares method, this is accomplished as follows: A large number of sets of injection conditions which are dispersed about the nominal injection point are selected. For each set, the Lunar Probe Ballistic Flight Simulation Program (LU-5) is used to compute the analytically correct value of the velocity required to impact the moon.

The sets of injection conditions, together with the correct velocity values are fed into the Least Squares Analysis Program (LS-2) which computes numerical values for the guidance constants. The LS-2 program allows flexibility in the terms which enter the guidance equations, thus easily allowing evaluation of the effect of adding and deleting terms in order to study the resulting influence on guidance accuracy. This derivation of the guidance constants completes the formulation of the guidance concept and we are now ready to perform an evaluation of guidance accuracy.

The accuracy evaluation is divided in two parts:

. . . . evaluation of guidance errors due to errors of guidance equations (discussed in this volume)

. . . . evaluation of guidance errors due to errors of guidance equipment (discussed in Volume IV)

For both of these phases, we return to the powered ascent simulation (AS-3) which was used in the derivation of the nominal trajectory. Now, however, the steering and thrust cutoff control of the missile simulates the behavior of the actual airborne guidance system being evaluated. A large number of powered "flights" are simulated with typical deviations from nominal missile behavior (mass flow rate, specific impulse, lift-off time, autopilot error). For evaluating the guidance equations, the injection conditions resulting from the powered ascent are used as inputs to the LU-5 program to evaluate the miss at the moon.

A detailed mathematical discussion of the simulation techniques is presented in Volume V.

4.3 Nominal Trajectories

4.3.1 General Remarks

It has been explained previously that the guidance system concept depends heavily on the idea of a nominal trajectory. For the purposes of the present study, a nominal trajectory may be defined as an ideal trajectory having the following characteristics precisely defined:

....launch time

....constant missile body axis azimuth

....predetermined missile body axis pitch program

....predetermined thrust acceleration vs. time

....predetermined thrust termination time

(The missile body axis azimuth and pitch are measured with respect to the inertial platform coordinate system as shown in Figure 3-3.)

All of these characteristics which define the nominal trajectory are selected so that the ballistic path followed by the vehicle after thrust termination will cause the vehicle to intercept the moon.

In an actual design problem, the nominal trajectory should satisfy other conditions besides simply impacting the moon. For example, it might be highly desirable to choose a nominal trajectory which maximizes the payload weight. However, since the present effort was not an overall vehicle study, but concentrated on guidance problems, these other considerations were de-emphasized in order to save time, although the simulation techniques used are completely adequate for solving these additional problems. As explained previously, the guidance system evaluation was carried out for both coplanar and non-coplanar cases. In what follows, we shall describe the nominal trajectories for both of these cases.

4.3.2 Thrust Acceleration Profile

In all the trajectory studies of this report, it was assumed that the vehicle has three stages of thrust. The basic thrust acceleration profile which was assumed is shown in Figure 4-2. This curve was stored in the IBM 704 powered ascent simulation (see Volume V - Section 3) used for all trajectory simulations. By appropriate changes in scaling (such as changing the vertical scale and shrinking or expanding the time scale), it was possible to simulate changes in fuel specific impulse and engine mass flow rate. These scale changes were used to insert off-nominal deviations in evaluating guidance system performance. It should be noted, however, that even for the nominal trajectories, these scale changes were used to save time in designing the nominal trajectories. Specifically, instead of carefully shaping the trajectory for optimum engine utilization, the engine performance was improved, when necessary, by rescaling the thrust acceleration curve of Figure 4-2.

Thus, the standard conditions of the nominal trajectories are offset somewhat from the curve of Figure 4-2, even when no off-nominal condition is being simulated. The precise values of these nominal offsets will be given later in this section.

4.3.3 Coplanar Nominal Trajectories

As will be described later, both low velocity and high velocity nominal trajectories were derived. This was done for the purpose of investigating the effect of injection velocity on guidance system performance.

In the low velocity case, the missile was flown in a true no-lift turn. This is done by continually aligning the missile thrust axis with the missile velocity relative to Earth-fixed coordinates. However, since the missile is launched in a vertical rise, it is necessary to insert a transient disturbance in the missile pitch attitude; otherwise, the missile will simply continue rising vertically until burnout. The shaping of the nominal no-lift turn can be controlled through controlling the nature of this transient disturbance. In this study the required transient disturbance was inserted as follows:

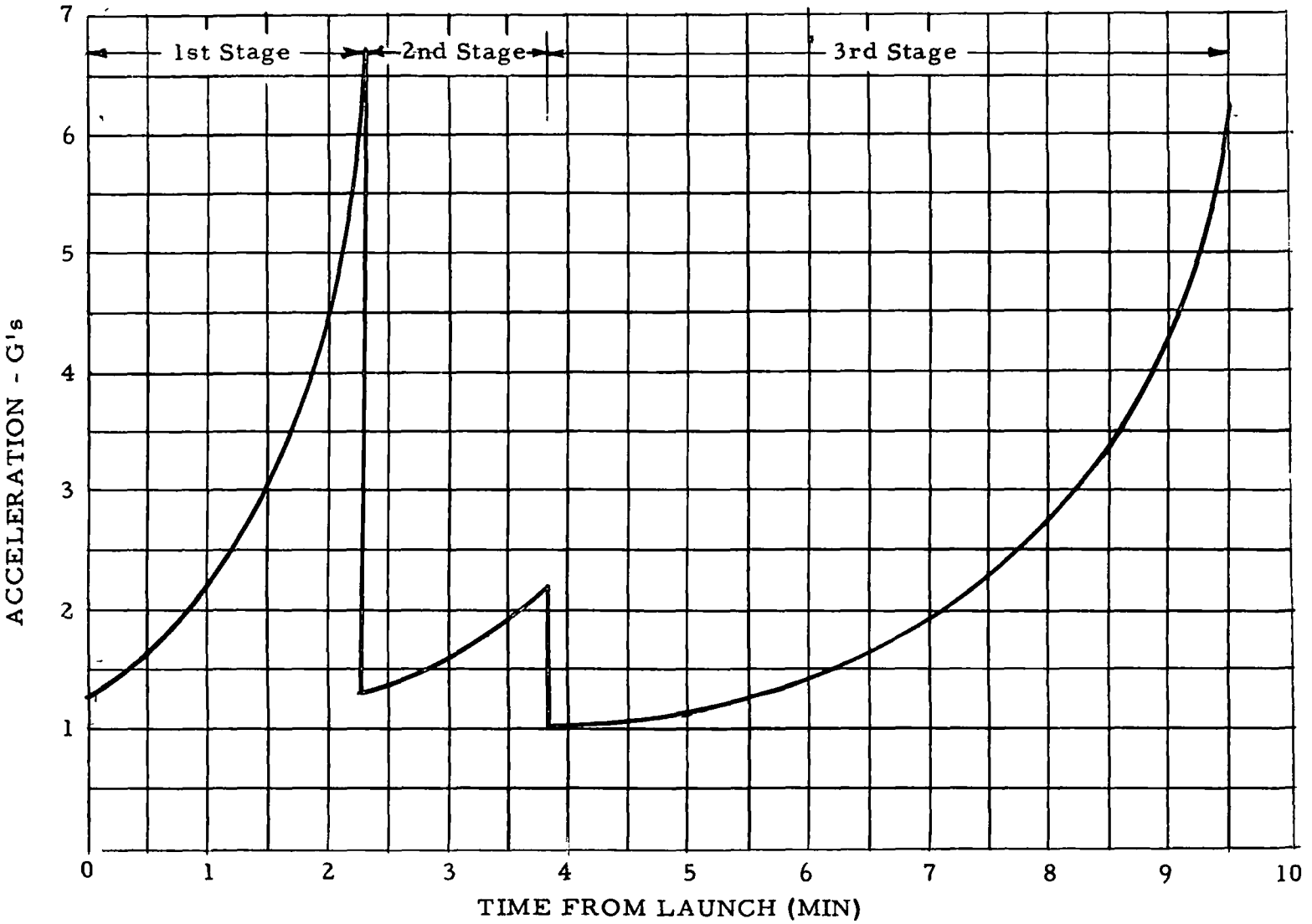

Figure 4-2 BASIC THRUST ACCELERATION PROFILE ASSUMED FOR TRAJECTORY STUDIES

The missile rises vertically for 15 seconds. Then the body axis is offset from the vertical by a small angle (known as the "kick angle"). This attitude, relative to the inertial platform, is held for a certain interval. At the end of this interval, the missile body axis (which we assume coincident with the thrust axis) is re-aligned with the velocity vector relative to Earth-fixed coordinates, and the missile continues on the characteristic curved path usually associated with a no-lift turn. A particular desired trajectory can be obtained by proper selection of the kick angle magnitude and the length of time it is held. When the desired no-lift turn has been found, the resulting profile of missile body axis pitch relative to the inertial platform is stored and becomes the pitch program for all trajectories.

For the low velocity coplanar nominal trajectory, it was desired that the missile at injection be at an altitude of about 300 miles and that its inertial velocity vector have a slightly positive path angle relative to the local horizontal. Trial simulations were made with various kick angle magnitudes and durations until the conditions were satisfied. The resulting missile pitch program is shown in Figure 4-3. In this plot, 90 degrees signifies the missile is vertical. It can be seen that the missile rises vertically for 15 seconds at which time a kick angle of 3.5 degrees is inserted. This is held for 10 seconds, and then the body axis is made coincident with the Earth-referenced velocity vector. One would normally expect a stepwise change in missile attitude as the swing of missile body axis into alignment with the velocity vector occurs. In general this would occur, and it is purely coincidental that on this trajectory at the moment the alignment should occur, the two vectors just happen to be coincident.

A physical feel for the nature of the low velocity coplanar nominal trajectory can be gained by examining Figure 4-4. This graph shows the missile's altitude as a function of the geocentric angle in inertial space measured from launch. The inertial velocity-time history in inertial space is also tabulated. The mass flow rate used is the same as that assumed in the thrust curve of Figure 4-2, but the specific impulse is assumed to be 5% above that used in Figure 4-2.

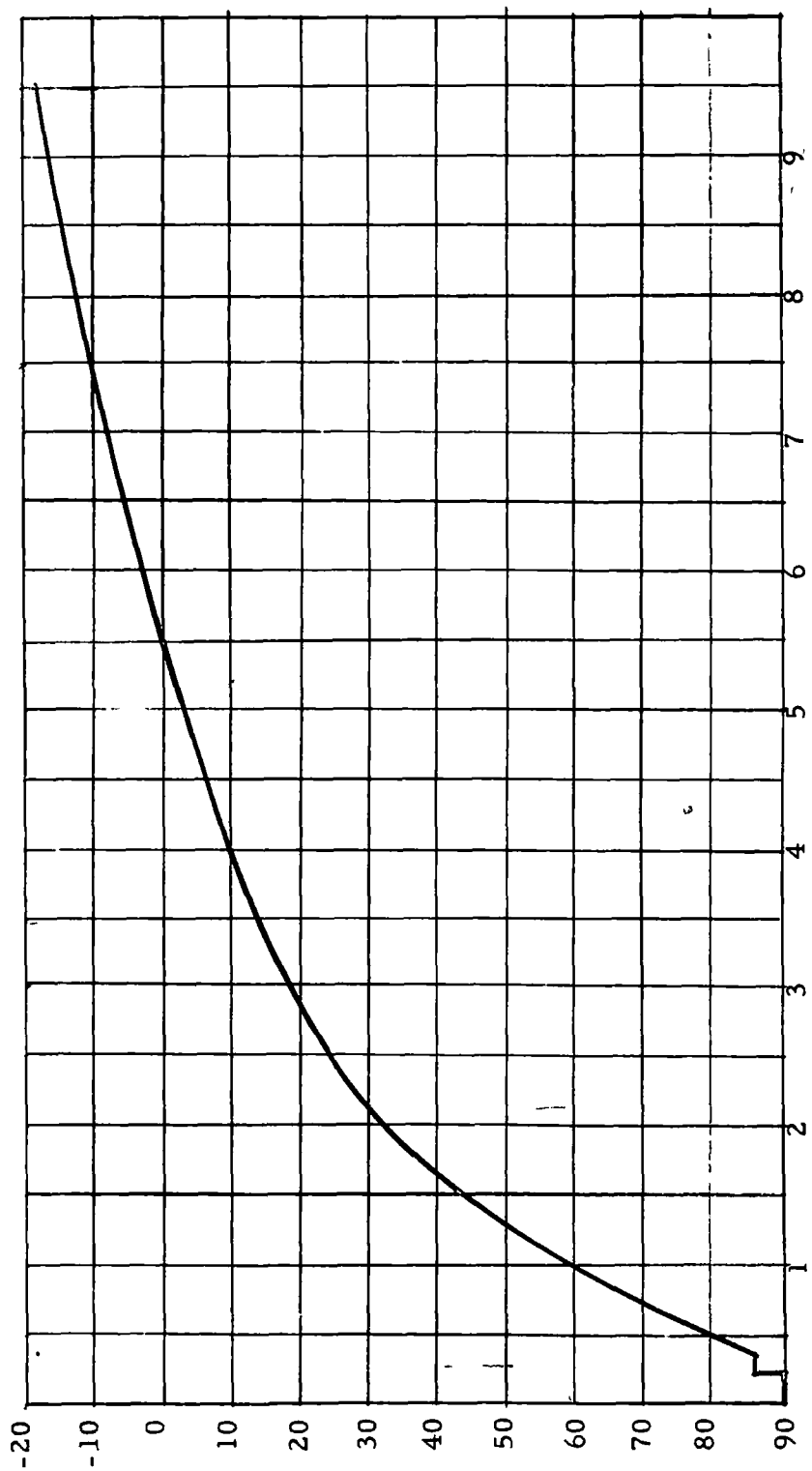

Figure 4-3 MISSILE BODY AXIS PITCH PROGRAM
USED FOR TRAJECTORY SIMULATIONS

TIME (SEC)	60	120	180	240	300	360	420	480	540	570
INERTIAL VELOCITY (FT/SEC)	2 276	7,232	11,991	14,801	16,610	18,939	21,920	26,103	33,024	37,338
$\dot{x}$ (FT/SEC)	1,903	6,424	11,334	14,480	16,549	18,936	21,906	25,644	31,919	35,834
$\dot{z}$ (FT/SEC)	1,247	3,324	3,912	2,966	1,419	-331	-2,376	-4,868	-8,468	-10,489

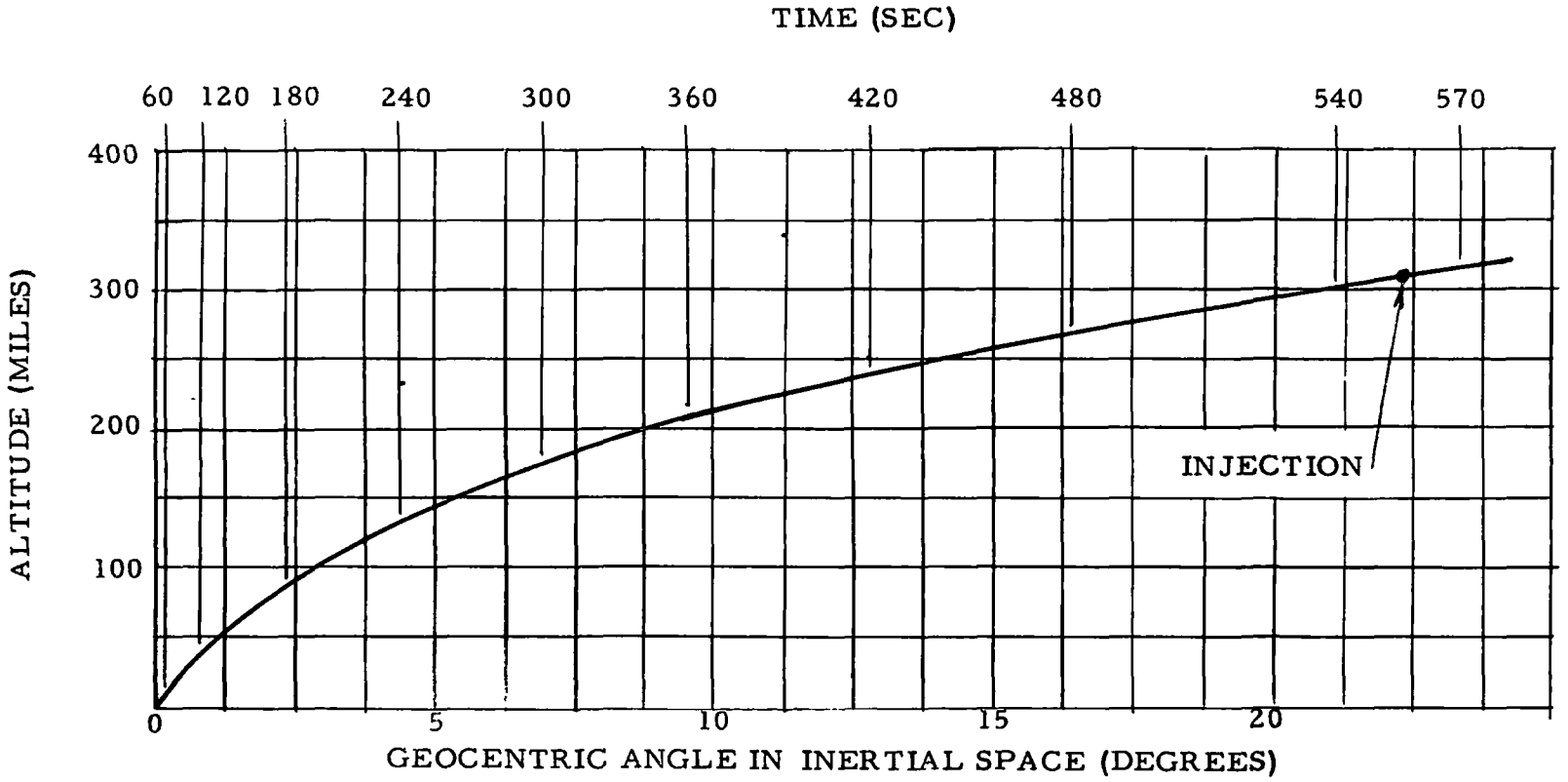

Figure 4-4 COPLANAR LOW VELOCITY NOMINAL TRAJECTORY
(RUN NO. 15)

CONFIDENTIAL
CONFIDENTIAL

For the coplanar high-velocity nominal trajectory, in order to save analysis time a true no-lift turn was not generated. Instead, the missile pitch program shown in Figure 4-3 (which generates a true no-lift turn for the low velocity trajectory) was used. The increase in velocity was obtained by raising the specific impulse 3% above that used for the low velocity nominal trajectory, i.e., 8% above that used in developing the thrust acceleration profile of Figure 4-2. The coplanar high velocity nominal trajectory is plotted in Figure 4-5 which shows altitude as a function of geocentric angle in inertial space and also gives a tabulation of the inertial velocity vs. time profile.

A summary tabulation of both coplanar nominal trajectories is given in Figure 4-6. In examining this table, it is helpful to recall some of the basic assumptions used in deriving these trajectories:

....The moon's orbital plane is taken as coincident with the Earth's equatorial plane.

....The vehicle is launched from an equatorial site.

....The platform inertial coordinate system is aligned so that the $\bar{Z}$ axis is vertical at launch and the $\bar{X}-\bar{Z}$ plane coincides with the Earth's equatorial plane. Its origin is at the launch point in inertial space.

In view of these assumptions, it is clear that all initial position and velocity components are zero, except for $\dot{\bar{X}}$ which reflects the Earth's eastward rotation. Actually, the value of $\dot{\bar{X}}$ used for initial velocity is more typical of a launch from approximately the latitude of Cape Canaveral than from the equator.

It is interesting to note that an increase of only 3% in specific impulse in going from the low velocity to the high velocity case is sufficient to cut the ballistic flight time by more than 50%.

TIME (SEC)	64.2	121.2	180	242	301	361.2	421.8	480	544 2	570
INERTIAL VELOCITY (FT/SEC)	2,570	7,666	12,366	15,295	17,126	19,501	22,590	26,701	33,950	38,271
$\dot{x}$ (FT/SEC)	2,099	6,769	11,632	14,951	17,038	19,501	22,493	26,321	32,947	36,890
$\dot{z}$ (FT/SEC)	1,481	3,597	4,196	3,226	1,729	-9 8	-2,079	-4,479	-8,188	-10,181

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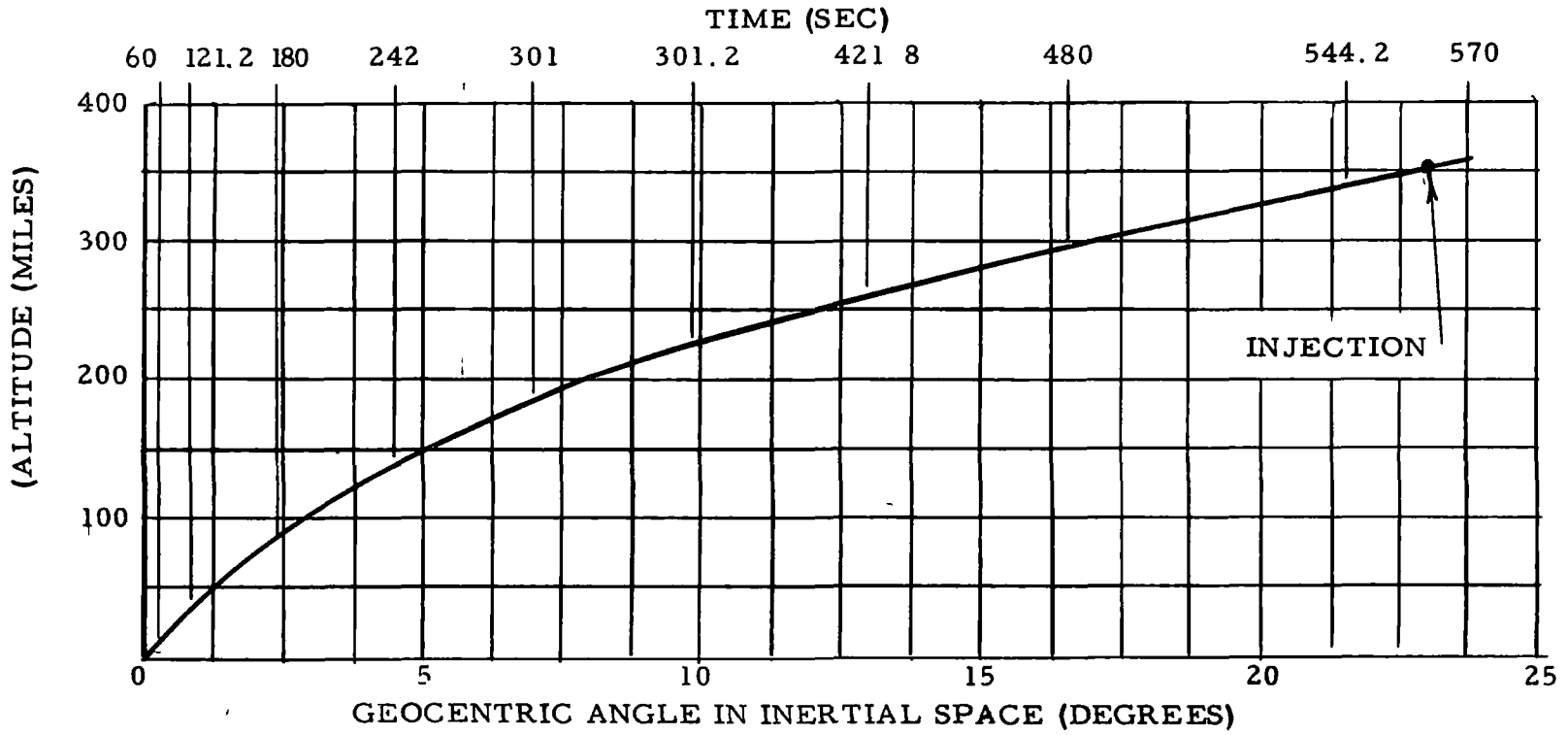

Figure 4-5 COPLANAR HIGH VELOCITY NOMINAL TRAJECTORY
(RUN NO. 21)

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COPLANAR LUNAR IMPACT MISSION NOMINAL TRAJECTORIES

	Low Velocity	High Velocity
Run No.	15	21
Launch Conditions		
X_o	0	0
Z_o	0	0
$\dot{X}_o$	14.16 miles/min	14.16 miles/min
$\dot{Z}_o$	0	0
Profile Data		
Specific Impulse	5% above Figure 4-2	8% above Figure 4-2
Mass Flow Rate	As in Figure 4-2	As in Figure 4-2
Pitch Profile	As in Figure 4-3	As in Figure 4-3
Ascent Trajectory	As in Figure 4-4	As in Figure 4-5
Injection Conditions		
Time	9.295836 min	9.308335 min
Geocentric Angle	22.33906°	22.79635°
Velocity	35,155.208 ft/sec	36,138 538 ft/sec
Altitude	310.384 miles	354.44 miles
Path Angle	7.0°	8.0°
X	1622.736 miles	1671.272 miles
Z	3948.970 miles	3976.508 miles
$\dot{X}$	384.6961 miles/min	397.0814 miles/min
$\dot{Z}$	-107.7125 miles/min	-104.7486 miles/min
Ballistic Flight Data		
Time of Flight	64.13890 hrs.	30.78901 hrs.
Sweep Angle	156.5114°	134.7828°

Figure 4-6 - COPLANAR LUNAR IMPACT MISSION NOMINAL TRAJECTORIES

4.3.4 Non Coplanar Nominal Trajectory

In deriving a three-dimensional nominal trajectory for a lunar mission, there are 2 classes of problems not directly associated with guidance which must be taken into account when planning actual flight operations:

- ....Optimization of trajectory from the viewpoint of available engine performance

- ...Selection of launch time, direction, etc., to be in accord with moon's motion as given in the lunar ephemeris.

In order to arrive at a quick evaluation of the guidance concept, neither of these considerations were given prime importance in designing the non-coplanar nominal trajectory for the present study. It should be noted however, that the Arma Space Research Group has investigated these two problems and data is available concerning the analytical techniques and numerical results. To save time, the pitch vs. time profile was made identical with that used for the coplanar trajectories, i.e., as given in Figure 4-3, even though this results in a non-coplanar nominal trajectory which is not a true no-lift turn. The launch site latitude was taken as 30 degrees North. The platform was aligned so that at the launch site the Z axis was vertical and the X axis had an azimuth orientation of 80 degrees measured clockwise from North. At launch, the inertial coordinate system was cast free in inertial space and the missile body axis was kept in the $X-Z$ plane. The mass flow rate was the same as that of Figure 4-2, but the specific impulse was 1.08 times the specific impulse used in the thrust profile of Figure 4-2.

A word should be said about the method of determining the injection conditions for the non-coplanar nominal trajectory. First, the declination of the moon at nominal impact time is selected. (For the present study nominal impact declination was taken as -23 degrees, and assumed to be maximum negative declination for that month.) Next, a powered ascent with the conditions outlined above is simulated. In addition to generating the trajectory of the missile, the IBM 704 simulation performs the following supplementary

calculation: at each point in the powered flight (after sufficient velocity has been acquired to reach the radius of the moon's orbit in ballistic flight), a computation is made of the declination the vehicle would have when it reached the radius of the moon's orbit assuming thrust is terminated at that point in the powered flight. That point in the powered flight which yields a declination equal to the desired impact declination is taken as the nominal injection point. The details of this calculation are in Volume V Section 2.

As indicated above, for an actual flight, it is now necessary to do further work to relate the launch time to the actual lunar ephemeris. (This process will, in general, require a slight change in the impact declination and in the launch time to allow the nominal trajectory to "mesh in" with the lunar ephemeris.) However, in the present study this was not done. Instead, a "pseudo-ephemeris" was assumed, i.e., it was assumed that the moon would be at the desired impact declination (-23 degrees) and at the same right ascension as the vehicle when the vehicle reached the impact declination.

In Figure 4-7 is shown a Mercator projection of the non-coplanar nominal trajectory which was used in this study. Since the trajectory was not related to the lunar ephemeris, only the absolute launch latitude is significant. The absolute launch longitude does not enter, only the change in longitude from the launch site being significant. In spite of this, Figure 4-7 is plotted starting from the coast of Florida since a trajectory "meshed in" with the ephemeris would appear quite similar to this. Figure 4-7 also includes a tabulation of altitude, inertial velocity magnitude, and inertial velocity path angle relative to the local horizontal as functions of time from launch.

Information summarizing the characteristics of the non-coplanar nominal trajectory follows:

Identification	Run No. 210
Engine Characteristics	
Mass Flow Rate	As in Figure 4-2
Specific Impulse	8% above Figure 4-2
Body Axis Pitch Profile	As in Figure 4-3
Launch Conditions	
Latitude	30 deg. North
Z Axis	Along local vertical
X Axis	80 deg. Azimuth
Injection Conditions	
Time from Launch	9.327 min.
Latitude	30.969 deg.
Right Ascension Increment	26.688 deg.
Longitude Increment	24 35 deg
Altitude	357.834 mi.
Inertial Velocity	36372.24 ft/sec.
Velocity Path Angle	8.151 deg.
Velocity Azimuth	94.035 deg.
X	1684.145 mi.
Y	-22.596 mi.
$\dot{Z}$	15.696 mi.
$\dot{X}$	399.579 mi/min
$\dot{Y}$	-2.090 mi/min
$\dot{Z}$	-105.6705 mi/min
Ballistic Flight Data	
Flight Time	28.6965 hr.
Sweep Angle	132.269 deg.
Impact Declination	-23 deg.

TIME MIN.	1	2	3	4	5	6	7	8	9	CUTOFF 9.327
ALTITUDE MI.	7	35	76	130	180	228	264	300	335	358
VELOCITY FT/SEC	2478	7777	11980	15035	16885	19676	22740	26400	32573	36372
PATH ANGLE (DEG)	39.37	28.62	23.32	17.64	13.35	9.71	7.82	7.02	7.45	8.15

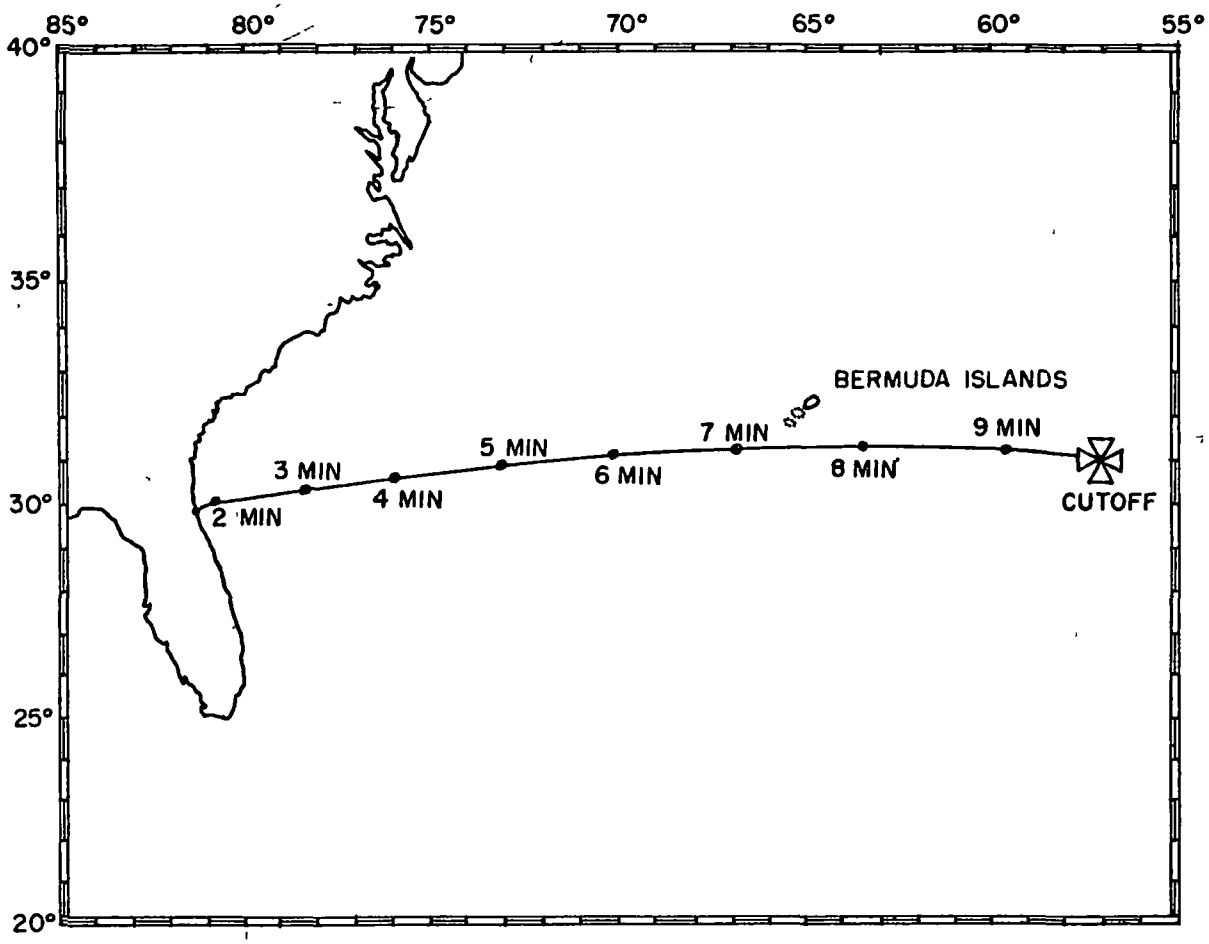

Fig 4-7 Mercator Projection of Non-Coplanar Nominal Trajectory

4.4 Guidance System Performance

4.4.1 General Remarks

In this section, we present detailed quantitative results concerning the expected performance of the guidance system which has been formulated in this volume. We are here concerned exclusively with the effectiveness of the system concept assuming perfect guidance equipment (gyros, accelerometers, etc.). Clearly, an overall system evaluation must include these latter considerations. A detailed description of analysis of the guidance equipment and its expected performance is presented in Volume IV of this report.

In this section, the performance results are presented in the order in which they were derived during the study program. Thus, we first present the results of coplanar trajectory simulations and follow this with the results of non-coplanar trajectory simulations. The general plan of attack was to first study the coplanar case in order to gain familiarity with the thrust termination guidance problem without being concerned with the azimuth steering problem or other complexities of the non-coplanar problem. In the coplanar work, several different polynomial expansions for χ_T were investigated as well as the effect of high and low velocity nominal trajectories. The information gained from the coplanar investigation was used to guide the non-coplanar investigation. Thus, in the non-coplanar case only one formulation of the polynomial expansions was tried. Also, only a single (high velocity) non-coplanar nominal trajectory was investigated. Non-coplanar runs on low velocity trajectories should be investigated, but time did not permit this to be done.

4.4.2 Results of Coplanar Trajectory Simulations

All of the guidance effectiveness results presented here were derived from a two-step simulation procedure:

....simulation of the powered ascent of the missile from launch under control of the guidance system until the moment of injection. This yields complete data on the position, velocity, and time of injection.

....starting from these injection conditions, a simulation of the free ballistic flight allows an evaluation of miss at the moon and the error in $\dot{X}$ (which is the controlling quantity in determining thrust cutoff) at the moment of injection.

The evaluation data is presented on a number of charts, a typical one being Figure 4-8. The rerun no. at the top of the chart refers to conditions concerning the polynomial expansions and the nominal trajectory which are held fixed for every trajectory in this rerun. Thus, for example, rerun no. 4 signifies the following:

....nominal trajectory - low velocity coplanar
....terms in expansion for $\dot{X}_r$ - all first and second order terms

....method of deriving constants - Taylor Series.

In the detailed discussion following, we shall point out the significance of each rerun number as we come to it.

Each horizontal line on the chart summarizes a single trajectory. A brief description of each item listed follows:

Run Number - This number identifies the off-nominal deviations simulated. The same run number is used in different re-run charts to identify a given set of deviations from nominal.

Off Nominal Deviations - As discussed earlier, the acid test of the guidance concept is its behavior when confronted with off nominal deviations in missile performance. The entries in the flow rate and specific impulse (specific thrust) columns indicate percentage deviations from the

flow rate and specific impulse used to derive the standard acceleration profile of Figure 4-2. However, as explained previously, the nominal trajectory itself has a specific impulse which differs somewhat from that of Figure 4-2. Thus, in Figure 4-8, the nominal trajectory is run no. 15, which has a specific impulse 5% above the standard acceleration profile of Figure 4-2. (Note that for every rerun the nominal trajectory is clearly indicated.) In run 17, the specific impulse is listed as 10%. This indicates 10% above the standard profile of Figure 4-2, but the significant quantity, i.e., the deviation from nominal as far as guidance performance is concerned, is 5% above the specific impulse of the nominal trajectory, run no. 15. It is important to bear this interpretation in mind.

In interpreting the flow rate column, a similar difficulty does not occur, since every nominal trajectory used a mass flow rate identical with that of the standard profile, Figure 4-2.

The column lift-off delay refers to the time interval between the instant the system starts full operation in inertial space and the instant the missile leaves the launching pad. As discussed in Section 3.7, this delay is intimately related to the operational concept for dealing with delays in count-down procedure.

The column pitch offset refers to an error in the pre-programmed pitch profile, such as might be caused by a drifting rate gyro used for damping, or a similar source which tends to produce a pitch attitude error.

Injection Conditions

Time (Min): This is the time from lift-off to injection. In the case of a non zero lift-off delay, the actual total operating time of the guidance system is the time listed in this column plus the delay in lift-off.

Down Range Angle (Deg.): The geocentric angle in inertial space swept by the vehicle between the start of inertial system operation and injection. In the case of a delayed lift-off, this includes the angle swept by the missile while on the launch pad during the lift-off delay.

Velocity (ft/sec): The total injection velocity in inertial space.

Path Angle (Deg.): The angular attitude of the inertial injection velocity relative to the local horizontal at injection.

X , Z · The displacement of the vehicle in the system's inertial coordinate system measured from the start of inertial navigation to injection.

$\dot{X}$, $\dot{Z}$ · The inertial velocity at injection expressed in the system's inertial coordinates.

Ballistic Flight Data

Time of Flight (hrs.): The ballistic flight time from injection until the radius of the moon's orbit is reached, ignoring the effect of lunar gravity.

Sweep Angle (Deg.): The geocentric angle swept by the vehicle between injection and reaching the radius of the moon's orbit (ignoring effect of lunar gravity).

Guidance Equation Performance

Errors in $\dot{X}$ (mi/min): The difference between the value of $\dot{X}$ at cutoff and the value of $\dot{X}$ which would give a perfect impact, given the value of the other injection variables X , Z , $\dot{Z}$, t . This error evaluates the performance of the polynomial expansion for $\dot{X}_r$. A positive value indicates that $\dot{X}$ at cutoff was greater than the $\dot{X}$ required for impact.

Miss Along Orbit Unfocused (mi): The distance between the vehicle and the center of the moon at the moment the vehicle reaches the radius of the moon's orbit, ignoring the effect of lunar gravity. A positive value of miss indicates that the vehicle would have crossed the moon's orbit "ahead" of the moon.

Distance of Closest Approach Unfocused (mi)· In general, since the vehicle orbit does not cross the moon's orbit at right angles, the distance of closest approach is somewhat smaller than the miss along the moon's orbit, even when lunar gravity effects are ignored. This is the quantity listed in this column.

Distance of Closest Approach Focused (mi): The distance of closest approach of the vehicle to the center of the moon including the effect of lunar gravity.

In the remainder of this section, we present a discussion of each of the reruns of coplanar trajectory simulations.

Rerun No. 4

Summary Chart: Figure 4-8 (2 sheets)

Nominal Trajectory: Low Velocity Coplanar

Terms in Polynomial Expansion for $\dot{X}_F$: $\Delta X, \Delta Z, \Delta \dot{Z}$ $\Delta t, \Delta X^2, \Delta X \Delta Z, \Delta X \Delta \dot{Z}, \Delta X \Delta t, \Delta Z^2, \Delta Z \Delta \dot{Z},$ $\Delta Z \Delta t, \Delta \dot{Z}^2, \Delta \dot{Z} \Delta t, \Delta t^2$.

Method of Deriving Constants: Taylor Series

This was the first actual evaluation of the guidance concept carried out during this study. Therefore, there was no previous experience upon which to base the selection of which terms should be in the expansion for $\dot{X}_F$. Hence, all first and second order terms were chosen. This rerun was the only attempt to choose the constants on the basis of the Taylor series approach (see Section 3.6). All other reruns used the least squares method. The "no solution" entries in the column "error in $\dot{X}$" are mostly explained by the fact that the iterative procedure for computing $\dot{X}_F$ was not yet completely developed at this point in the study, and hence did not converge when starting with large errors.

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 4

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
15	Nominal Trajectory 0	5	0	0	9.295836	22.33906	35,155.2	310.3835	6.697141	1622.736	3948.970	384.6961	-107.712	64.13800	156.5114	0	0	0	0
17	0	10	0	0	9.145290	22.18365	35,154.6	373.4488	8.557196	1635.831	4011.754	388.2406	-94.1150	47.36377	145.9351	No Soln.	-6300.95	-5584.94	-4725.60
19	0	9	0	0	9.174140	22.20969	35,136.4	361.0832	8.188524	1632.980	3999.562	387.3815	-96.7370	50.28488	148.1362	No Soln.	-3709.87	-3215.08	-2306.48
21	0	8	0	0	9.203594	22.23810	35,126.9	348.5988	7.817974	1630.238	3987.196	386.5940	-99.4052	53 48422	150.3292	No Soln.	-1780.16	-1499.75	-696.465
23	0	7	0	0	9.233559	22.26836	35,126.0	335.9834	7.445498	1627.563	3974.660	385.8758	-102.117	56.98318	152.5049	-.1582	-600.960	-488.540	-78.617
24	0	6.5	0	0	9.248921	22.28521	35,130.0	329.6353	7.059030	1626.325	3968.307	385.5553	-103.498	78.73305	153.5395	-.0446	-227.361	-181.184	-10.287
25	0	6	0	0	9.264392	22.30245	35,135.9	323.2511	7.072103	1625.096	3961.911	385.2487	-104.888	60.55093	154.5660	-.0063	-41.2748	-32.1678	-.299
28	0	4	0	0	9.328041	22.37898	35,184.9	297.3797	6.320996	1620.536	3935.814	384.2280	-110.596	67.32627	158.2352	.0038	49.04253	34.63164	.276
29	0	3.5	0	0	9.344403	22.40006	35,203.8	290.8236	6.132459	1619.486	3929.156	384.0271	-112.061	68.59859	158.9801	.0222	326.2290	225.4505	11.161
30	0	3	0	0	9.360849	22.42143	35,225.5	284.2271	5.943438	1618.435	3922.954	383.8502	-113.541	69.54386	159.6261	.0577	942.8200	640.7642	86.16
32	0	2	0	0	9.393973	22.46492	35,277.9	270.9082	5.564040	1616.323	3908.916	383.5709	-116.544	70.17188	160.5715	No Soln.	3625.436	2435.722	1024.228
34	0	1	0	0	9.427511	22.51004	35,342.5	257.4239	5.183337	1614.238	3895.185	383.3951	-119.610	68.92750	161.0138	No Soln.	8507.055	5843.836	3983.867
36	0	5	0	0	9.461503	22.55703	35,419.9	243.7707	4.801762	1612.195	3881.251	383.3279	-122.743	66.09729	160.9863	No Soln.	15070.89	10851.09	8967.211
37	0	5	0	.2	9.291380	22.38005	35,162.0	306.0339	6.555443	1623.905	3943.786	384.4255	-108.959	65.52499	157.2256	-.0025	-27.1901	-19.7617	-.098
39	0	5	0	.1	9.293592	22.35949	35,158.3	308.2090	6.626288	1623.317	3946.380	384.5585	-108.335	64.84934	156.8758	-.0022	-23.2861	-17.0995	-.068
42	0	5	0	-.1	9.298140	22.31892	35,152.3	312.5606	6.76825	1622.175	3951.554	384.8354	-107.090	63.42960	156.1425	.0002	2.999082	2.248524	.001
44	0	5	0	-.2	9.300461	22.29883	35,149.9	314.7368	6.839290	1621.615	3954.136	384.9793	-106.471	62.69388	155.7608	.0016	13.50284	10.22861	.025

Figure 4-8
(No. 1 of 2 Sheets)

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO.

I.D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
65	-5	5	0	0	9.735705	23 92514	35.362 4	257.3470	5.386719	1709.909	3854.062	380.9941	-127.763	66.07979	159.7918	No Soln.	15824.05	11397.05	9496.37
69	-3	5	0	0	9 556383	23 26617	35,242 5	278.9152	5.880026	1673.990	3893.287	382.1868	-119.668	70.44366	160.0005	No Soln.	3956.364	2644.834	1170.155
71	-2	5	0	0	9 468466	22 94928	35,201 2	289.4967	6.145481	1656.557	3912.230	382.9335	-115.642	69.91255	159.3143	.0592	994.8178	671.6270	93.474
73	-1	5	0	0	9 381601	22 64005	35,171.6	299.9800	6.418214	1639.454	3930.789	383.7659	-111.652	67.66692	158.1348	-.0047	-63.4958	-44.5850	-.45
74	-.5	5	0	0	9 338576	22 48860	35,162.1	305.2012	6.557250	1631.055	3939.933	384.2230	-109.676	65.96274	157.3541	-.0038	-43.3657	-31.3039	-.23
76	1	5	0	0	9 210830	22.04507	35,149.9	320.7544	6.983057	1606.345	3966.857	385.7091	-103.798	60.20732	154.6272	-.0112	-70.1216	-54.8867	-.89
78	2	5	0	0	9 126878	21 75746	35,157.0	330.9289	7.273822	1590.184	3984.320	386.8155	-99.9194	56.23443	152.5614	-.2367	-775.872	-635.845	-134.85
80	3	5	0	0	9 043791	21 47651	35,176.4	341.0531	7.573595	1574.334	4001.491	388.0219	-96.0466	52.41997	150.3591	No Soln.	-2385.86	-2029.89	-1155.51

Figure 4-8
(No. 2 of 2 Sheets)

As mentioned earler (see Section 3.6), the Taylor series method of deriving constants puts very heavy emphasis on a small region around the nominal trajectory. Hence, on this rerun, the nominal trajectory (run 15) has zero miss.

It can be seen by comparing "distance of closest approach focused" with "distance of closest approach unfocused" that the gravitational field of the moon has considerable effect in this rerun in reducing the miss. In later reruns based on a high velocity nominal trajectory, it will be seen that this focusing effect is considerably less. Actually, the unfocused miss due to the guidance concept presented here should be combined with the unfocused miss due to the guidance equipment before the focusing effect is applied since the focusing effect is non-linear. This matter is discussed more fully in Volume IV.

In this rerun, the off-nominal deviations were applied singly, i.e., were not combined. In later reruns, we shall exhibit the performance of the system when confronted with off nominal deviations of several types simultaneously. For Rerun 4, lift off delay conditions were not investigated.

Since engine performance deviations are of prime importance, we tabulate (in summarized form) the accuracy of guidance versus deviations in mass flow rate and specific impulse:

Rerun 4 - Mass Flow Rate Deviations from Nominal

Run No.	Mass Rate Dev.%	Dist. of Closest. App. (Unfocused) Mi.
65	-5	15,824
69	-3	3,956
71	-2	994
73	-1	-63
15 (Nom)	0	0
74	-.5	-43
76	+1	-70
78	+2	-775
80	+3	-2,385

Rerun 4 - Specific Impulse Deviations from Nominal

(NOTE: Nominal I_{sp} is 5% above that of Figure 4-2)

Run No.	Sp. Impulse Dev. %	Dist. of Closest App. (Unfocused) Mi.
36	-5	10,851
34	-4	5,843
32	-3	2,435
30	-2	641
29	-1.5	225
28	-1	34
15 (Nom.)	0	0
25	+1	-32
24	+1.5	-181
23	+2	-488
21	+3	-1,500
19	+4	-3,215
17	+5	-5,585

From the above tabulations, it can be seen that, roughly speaking, the following off nominal deviations are allowable:

Mass Flow Rate: -1 1/2% to +1 1/2%

Specific Impulse: -1 1/2% to +1 1/2%

These figures assume an allowable error of about 200 miles. Actually, these are somewhat optimistic figures, since as mentioned above, off nominal deviations of different types were not applied simultaneously.

The results of this rerun are typical of what may be expected when the constants of the polynomial expansion are derived by the Taylor series method, i.e., extremely good results close to nominal conditions, but rapid increase in errors as off-nominal deviations are increased.

Rerun No. 6

Summary Chart: Figure 4-9 (2 sheets)

Nominal Trajectory: Low Velocity Coplanar

Terms in Polynomial Expansion for $\dot{\chi}_r$: $\Delta x, \Delta z, \Delta \dot{z}, \Delta t, \Delta x^2, \Delta x \Delta z, \Delta x \Delta \dot{z}, \Delta x \Delta t, \Delta z^2, \Delta z \Delta \dot{z}, \Delta z \Delta t, \Delta \dot{z}^2, \Delta \dot{z} \Delta t, \Delta t^2$

Method of Determining Constants: Least Squares

Input Data for Least Squares Calculation - It was mentioned earlier that derivation of the constants by the least squares method requires a large number of sets of injection conditions dispersed about the nominal injection point. The selection of these dispersed injection conditions is a critical factor in the procedure. For this rerun, the selection was made as follows: The trajectory simulations of the previous rerun (rerun no. 4) were used as the source of dispersed injection conditions for rerun no. 6 On each trajectory from rerun no. 4, the injection point, a point shortly before injection, and a point shortly after injection (assuming burning continues) were used as the input conditions for the least squares analysis of rerun no. 6.

Standard Deviation of $\dot{\chi}_r$ from Least Squares Calculation: 0.0694 mi/min. (Note: In performing the least squares calculation of constants, the LS-2 program described in Volume V utilizes as inputs a large number of off-nominal injection points and the analytically correct value of $\dot{\chi}_r$ for each point. In addition to computing the best least squares values of the constants, the program also determines the standard deviation of the errors between the analytically correct $\dot{\chi}_r$ and the $\dot{\chi}_r$ computed by the polynomial expansion. This is the quantity given above, and gives a rough evaluation of the polynomial expansion. It is not a complete evaluation, however, since it is based only on the data used to derive the polynomial, and also does not give the miss at the moon but only the velocity error.)

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 6

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI.)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS.)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
15	Nominal Trajectory 0	5	0	0	9.295908	22.33944	35,155.8	310.3867	6.697268	1622 764	3948.962	384.7029	-107.716	64.05894	156.4861	.0083	79.16752	58.81903	1.577
17	0	10	0	0	9.152256	22.22008	35,217.7	373.8634	8.568788	1638.538	4011.097	388.8956	-94.4523	45.07827	144.6630	No Solution	-6216.77	-5597.13	-4829.12
19	0	9	0	0	9.178667	22.23343	35,177.7	361.3415	8.1964	1634 735	3999.124	387.8107	-96.9583	48.40215	147.1967	No Solution	-3214.45	-2827.39	-2006.21
21	0	8	0	0	9.206290	22.25228	35,151.9	348.7452	7 822865	1631.280	3986.928	386 8531	-99.5386	52.04025	149.6863	No Solution	1092.941	-933.047	-323.49
23	0	7	0	0	9.235055	22.27627	35,140.1	336.0609	7 448345	1628.141	3974.507	386.0217	-102.192	55 94100	152.0921	.0278	98.70814	81.14539	2.33
24	0	6.5	0	0	9.249915	22.29048	35,139.4	329.6857	7.260973	1626.709	3968.204	385.6529	-103.548	57.94499	153.2446	.0767	370.8415	298 2372	28.62
25	0	6	0	0	9 265026	22.30581	35,142.0	323.2827	7.073346	1625.340	3961.845	385 3113	-104.921	59.97852	154 3638	.0705	441.0407	346.2052	36.842
28	0	4	0	0	9.327524	22.37622	35,179.8	297.3570	6 319891	1620.338	3935.871	384.1755	-110.569	68.08893	158 4543	-.0594	-796.352	-555.207	-67.91
29	0	3.5	0	0	9.343513	22.39530	35,194.9	290.7862	6.130522	1619.145	3929.256	383.9358	-112.014	70.06283	159.3831	-.0874	-1367.28	-920.594	-172.04
30	0	3	0	0	9.359538	22.41438	35,212.2	284.1731	5.940571	1617.932	3922.603	383.7137	-113 470	71.92705	160.2589	-.1059	-1908.48	-1240.35	-289.67
32	0	2	0	0	9.391678	22.45253	35,254.0	270.8188	5.558727	1615.443	3909.183	383.3253	-116.417	74.94523	161 7829	- 1012	-2311.88	-1411.38	-343.81
34	0	1	0	0	9.423969	22 49079	35,304.6	257.2949	5.174805	1612.880	3895.608	383.0063	-119 410	76.53235	162.9383	No Solution	-972 875	-573.045	-57.32
36	0	0	0	0	9.456402	22.52916	35,363.9	243.5988	4.788956	1610.241	3881.876	382.7537	-122.448	76.27732	163.6705	No Solution	2822.542	1672.031	458.85
37	0	5	0	.2	9.290996	22.37800	35,158.3	306.0164	6.554638	1623.757	3943.828	384.3872	-108.939	66.01968	157.3753	-.0488	-545 023	-393.084	-36.68
39	0	5	0	.1	9.293497	22.35898	35,157.4	308.2042	6.626089	1623 280	3946 390	384.5491	-108.330	64.96472	156.9115	-.0136	-140.679	-103.124	-2.63
42	0	5	0	.-1	9.298262	22.31957	35,153 5	312.5664	6.768517	1622.222	3951.541	384.8477	-107.097	63.29303	156.0983	.0152	134.4503	100.9979	2.69
44	0	5	0	.-2	9.300529	22.29919	35,150.6	314.7402	6.839424	1621.641	3954.129	384.9861	-106.475	62.62149	155.7369	0098	81.35303	61.68776	1.03

Figure 4-9
(No 1 of 2 Sheets)

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 6 (Cont'd)

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
65	-5	5	0	0	9.729384	23.89081	35,305.9	257.1088	5.369717	1707.503	3854.868	380.4134	-127.440	76.65295	162.5733	No Solution	3053.736	1793.518	518.3357
69	-3	5	0	0	9.554817	23.25775	35,227.9	278.8508	5.876344	1673.392	3893.474	382.0384	-119.587	73.24735	160.7251	.0238	518.6025	328.2089	20.595
71	-2	5	0	0	9.468048	22.94705	35,197.2	289.4789	6.14462	1656.398	3912.278	382.8932	-115.620	70.60183	159.4996	.0104	178.9839	119.3119	2.944
73	-1	5	0	0	9.381634	22.64023	35,171.9	299.9822	6.418286	1639.467	3930.786	383.7691	-111.654	67.61995	158.1213	-.0009	-11.5240	-8.0982	-.01488
74	-.5	5	0	0	9.338608	22.48877	35,162.4	305.2021	6.557325	1631.067	3939.929	384.2262	-109.677	65.92145	157.3417	0	.0588944	.04254	0
75	+.5	5	0	0	9.253351	22.19218	35,152.7	315.6077	6.842094	1614.581	3957.954	385.2090	-105.756	62.02080	155.5363	.0228	176.8100	135.1729	5.042
76	+1	5	0	0	9.211238	22.04724	35,154.0	320.7748	6.983864	1606.503	3966.815	385.7509	-103,819	59.83233	154.4937	.0398	241.7550	190.1136	10.8357
78	+2	5	0	0	9.128404	21.76555	35,172.3	331.0069	7.276618	1590.775	3984.168	386.9743	-99,9985	55.16217	152.1267	-.0320	-97.6652	-80.9358	-2.396
80	+3	5	0	0	9.047568	21.49645	35,215.0	341.2524	7.579996	1575.800	4001.128	388.4245	-96,2437	50.39638	149.4177	No Solution	-1591.44	-1377.88	-665.659
84	+5	5	0	0	8.893170	21.00248	35,394.2	361.8155	8.214046	1548.616	4033.762	392.2307	-89,0293	41.47011	143.1726	No Solution	-9229.82	-8490.36	-7843.83

Figure 4-9
(No. 2 of 2 Sheets)

This rerun was identical in all respects with rerun no. 4, except that the constants were derived by the method of least squares instead of by the Taylor series method which was used in rerun no. 4.

Note that the nominal trajectory (run no. 15) now has an unfocused distance of closest approach of 58.8 miles, while in the Taylor series method (rerun no 4) the nominal trajectory showed zero miss. This is due to the fact that in the least squares method the nominal injection point is not weighted any more heavily than the dispersed injection points in deriving the constants.

For reruns based on the low velocity nominal trajectory, the variation of the sensitivity of miss at the moon to errors in $\dot{x}_r$ is very striking. Consider the following data for example:

Rerun no. 6

Run No.	Error in $\dot{x}$ (mi/min)	Dist. Closest App. Unfocused (Mi.)
23	.0278	81.15
69	.0238	328.21

Note that although the errors in $\dot{x}_r$ are almost identical, the misses are in a 4 to 1 ratio. This non-linearity is much less pronounced on those reruns based on high velocity nominal trajectories.

Once again in this rerun, the off nominal deviations were applied singly, and delays in lift-off time were not investigated.

We summarize below the guidance errors versus deviations in mass flow rate and specific impulse.

Rerun 6 - Mass Flow Rate Deviations from Nominal

Run No.	Mass Rate Dev. (%)	Dist. of Closest App. Unfocused (Mi.)
65	-5	1,793.5
69	-3	328.2
71	-2	119.3
73	-1	-8.1
74	-.5	0 04
15 (Nom.)	0	58.8
75	+.5	135.2
76	+1	190.1
78	+2	-80.1
80	+3	-1,377.9
84	+5	-8,490.4

Rerun 6 - Specific Impulse Deviations from Nominal

(Note· Nominal I_{sp} is 5% above that of Figure 4-2)

Run No.	Sp. Impulse Dev. (%)	Dist. of Closest App. Unfocused (Mi.)
36	-5	1,672.0
34	-4	-573.0
32	-3	-1,411.4
30	-2	-1,240.4
29	-1.5	-920.6
28	-1	-555.2
15 (Nom.)	0	58.8
25	+1	346.2
24	+1.5	298.2
23	+2	81.1
21	+3	-933.0
19	+4	-2,827.4
17	+5	-5,597.1

Clearly, an improvement in performance with mass flow rate deviations has been accomplished by switching from the Taylor series approach to the least squares approach. On the other hand, for reasons that are not entirely clear, performance with specific impulse variations has deteriorated. Assuming an allowable error of about 200 miles the following deviations from nominal are allowable:

Mass Flow Rate: -2 1/2% to +2 1/2%

Specific Impulse: -1/2% to +1/2%

Rerun No. 7

Summary Chart: Figure 4-10 (2 sheets)

Nominal Trajectory: High Velocity Coplanar

Terms in Polynomial Expansion for $\dot{X}_f$: $\Delta x, \Delta z, \Delta \dot{z}, \Delta t, \Delta x^2, \Delta x \Delta z, \Delta x \Delta \dot{z}, \Delta x \Delta t, \Delta z^2, \Delta z \Delta \dot{z}, \Delta z \Delta t, \Delta \dot{z}^2, \Delta \dot{z} \Delta t, \Delta t^2$

Method of Determining Constants: Least Squares

Input Data for Least Squares Calculation: Same as for Rerun No. 6, i.e., deviations from nominal of points taken from trajectories simulated in rerun no. 4 (specifically, all cut-off points plus a point on each trajectory shortly before and shortly after cutoff)

Standard Deviation of $\dot{X}_f$ from Least Squares Calculation:
0.1047 mi/min

This rerun was identical in all respects with rerun no. 6, except that it was based on the high velocity nominal trajectory instead of the low velocity nominal trajectory Notice that although the errors in $\dot{X}$ at cutoff are about the same order of magnitude as those in rerun no. 6, the miss at the moon is considerably smaller due to the reduced sensitivity of miss to velocity errors. This reduction in sensitivity is brought about by the increase in velocity.

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 7

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
21	Nominal Trajectory		0	0	9.308093	22.79505	36,136.7	354.4259	8.018082	1671.176	3976.533	397.0637	-104.738	30.80604	134.8013	-.0169	32.78253	31.54256	1.537
85	0	13	0	0	9.040001	22.00415	35,052.0	408.3996	9.620976	1636 350	4049.264	389.0516	-85.4187	45.98131	143.0653	No Soln.	-3583.24	-3206.78	-2451.56
87	0	12	0	0	9.094919	22.16727	35,259.0	398.1975	9.297455	1644.022	4035.141	390.6060	-89.2441	41.20820	140.8439	1.3091	-1217.03	-1121.14	-638.99
89	0	11	0	0	9.149720	22.33079	35,474.3	387.7397	8.976814	1651.558	4020.759	392.2179	-93.1066	37.62867	138.9538	.1368	-216.395	-202.898	-37.43
17	0	10	0	0	9.203933	22.49220	35,694.3	376.9820	8.657970	1658.763	4006.151	393.8514	-96.9887	38.84230	137.3377	-.0566	101.0521	95.8641	10.31
18	0	9.5	0	0	9.230448	22.57012	35,804.9	371.4671	8.498215	1662.093	3998.799	394.6650	-98.9272	33.67305	136.6250	-.0721	132.1616	125.9319	19.08
19	0	9.	0	0	9.256693	22.64682	35,915.7	365.8723	8.338505	1665.290	3991.407	395.4726	-100.865	32.62157	135.9673	.0631	118.0048	112.8644	16.56
23	0	7	0	0	9.358063	22.93635	36,356.5	342.6429	7.695568	1676.386	3961.548	398.6112	-108.605	29.29292	133.8116	.0207	-39.3975	-38.0798	-.48
24	0	6.5	0	0	9.382231	23.00285	36,465.9	336.6117	7.532718	1678.626	3954.048	399.3720	-110.531	28.62404	133.3733	.0306	-57.9840	-56.1504	-5.65
25	0	6	0	0	9.405952	23.06708	36,575.2	330.4959	7.369012	1680.661	3946.537	400.1254	-112.455	28.00270	132.9666	.0335	-63.2309	-61.3347	-7.07
15	0	5	0	0	9.452154	23.18920	36,794.4	318.0191	7.039284	1684.156	3931.477	401.6183	-116.300	26.87668	132.2317	.0213	-39.7044	-38.6256	-3.11
28	0	4	0	0	9.496968	23.30421	37,014.3	305.2325	6.707245	1686.986	3916.345	403.0933	-120.144	25.87956	131.5871	-.0192	35.08783	34.21683	2.67
92	0	8	0	.2	9.312551	22.88569	36,231.4	350.4410	7.894599	1675.915	3970.213	397.7082	-106.499	30.08720	134.2870	.0459	-86.8745	-83.7732	-11.10
94	0	8	0	.1	9.309978	22.83850	36,180.6	352.4159	7.955625	1679.401	3973.412	397.3508	-105.599	30.47251	134.5759	-.0201	38.32503	36.91361	2.11
97	0	8	0	-.1	9.306876	22.75522	36,099.6	356.4693	8.081895	1669.202	3979.578	396.8446	-103.912	31.08413	134.9623	.0524	-99.1877	-95.3523	-13.29
99	0	8	0	-.2	9.306333	22.71902	36,069.3	358.5464	8.147114	1667.489	3982.548	396.6943	-103.123	31.30125	135.0555	.0191	-358.795	-344.681	-136.80

Figure 4-10
(No. 1 of 2 Sheets)

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 7 (Cont'd)

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
100	-5	8	0	0	9.823276	24.81837	37,040 6	307.2349	6.986284	1790.723	3872.218	400.6948	-128.896	25.68895	130.8335	-2.0586	3630.032	3541.473	3333.104
104	-3	8	0	0	9.625688	24.04684	36,764 3	327.1582	7.379179	1746.538	3914.174	400 2235	-119.826	26.83743	131.5440	-.4135	751.9588	731.6001	531.533
106	-2	8	0	0	9.522107	23.63884	36,578.5	336.5648	7 584758	1722.393	3935.129	399.4546	-114.949	27.81104	132.3309	-.0564	104.9446	101.8493	19.35935
108	-1	8	0	0	9.415921	23.21955	36,366 2	345.6324	7.799221	1697.125	3955.961	398.3763	-109.883	29.11865	133.4202	.0495	-93.3217	-90.2455	-13.8304
109	-.5	8	0	0	9 362188	23.00779	36,253.0	350.0592	7.908020	1684.223	3966 281	397.7430	-107.317	29.91145	134.0759	.0300	-57.1087	-55.0989	-4.91703
110	+.5	8	0	0	9 253710	22.58218	36,018.1	358.7598	8.130610	1658.055	3986.716	396.3468	-102.144	31.81481	135.5951	- 0774	145 5655	139.6052	26.5028
111	+1	8	0	0	9.199351	22.36986	35,899.2	363.0132	8.244677	1644.889	3996 766	395.6115	-99.5556	32.94162	136 4495	- 1306	242.9540	232.1108	64.53624
113	+2	8	0	0	9.090862	21.94860	35,663 3	371.3250	8.482740	1618.566	4016.461	394.1246	-94.3724	35.61505	138.3310	-.1199	209 7876	198.4054	40.5803
115	+3	8	0	0	8.983201	21.53517	35,435.8	379.5023	8.737236	1592.544	4035.642	392.6760	-89.1989	38.97614	140.4227	.3427	-490.147	-456.695	-158.442
119	+5	8	0	0	9.774556	20.75291	35,046.8	395 6389	9.285683	1543.016	4072.098	390.3103	-79.1770	48.08991	144.8514	No Soln.	-6160.29	-5431.31	-4544.08

Figure 4-10
(No. 2 of 2 Sheets)

We summarize below the guidance errors versus deviations in mass flow rate and specific impulse.

Rerun No. 7 - Mass Flow Rate Deviations from Nominal

Run No.	Mass Rate Dev. (%)	Dist. of Closest App. Unfocused (Mi)
100	-5	3,541.5
104	-3	731.6
106	-2	101.8
108	-1	-90.2
109	-.5	-55.1
21 (Nom.)	0	31.5
110	+.5	139.6
111	+1	232.1
113	+2	198.4
115	+3	-456.7
119	+5	-5,431.3

Rerun No. 7 - Specific Impulse Deviations from Nominal

(Note: Nominal I_{sp} is 8% above that of Figure 4-2)

Run No.	Sp. Impulse Dev. (%)	Dist. of Closest App. Unfocused (Mi)
28	-4	34.2
15	-3	-38.6
25	-2	-61.3
24	-1.5	-56.1
23	-1	-38.1
21 (Nom.)	0	31.5
19	+1	112.9
18	+1.5	125.9
17	+2	95.9
89	+3	-202.9
87	+4	-1,121.1
85	+5	-3,206.8

It can be seen that the high velocity nominal trajectory has not significantly affected performance with off-nominal mass flow rates, but has brought about great improvement with specific impulse deviations. Specifically, if we consider a 200 mile miss as permissible, then the allowable limits are roughly:

Mass Flow Rate: -2 1/2% to +2%

Specific Impulse: -4% to +3%

Rerun No. 10

Summary Chart: Figure 4-11 (1 Sheet)

Nominal Trajectory: Low Velocity Coplanar

Terms in Polynomial Expansion for $\dot{\chi}_r$: $\Delta X, \Delta Z, \Delta\dot{Z}, \Delta t, \Delta X^2, \Delta X\Delta Z, \Delta X\Delta\dot{Z}, \Delta Z^2, \Delta Z\Delta\dot{Z}, \Delta\dot{Z}^2, \Delta\dot{Z}\Delta t, \Delta X^3, \Delta X^2\Delta Z, \Delta\dot{Z}^3, \Delta X\Delta\dot{Z}^2, \Delta Z\Delta\dot{Z}^2, \Delta\dot{Z}^4$

Method of Determining Constants: Least Squares

Input Data for Least Squares Calculation: Same as reruns 6 & 7 (i.e., based on points taken from trajectories simulated in rerun no. 4)

Standard Deviation of $\dot{\chi}_r$ from Least Squares Calculation: .0249 mi/min.

Up to this point, the polynomial expansion for $\dot{\chi}_r$ included all first and second order terms. On the basis of the results obtained from reruns No 6 and No. 7, an investigation was made on a more rational basis. Plots were made showing the dependence of $\dot{\chi}_r$ on the variables entering the expansion. On the basis of this investigation, it was found that certain terms which had entered earlier reruns could be dropped, but conversely it appeared that other terms should be added. A summary of these changes is as follows:

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 10

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
	Nominal Trajectory																		
15	0	5	0	0	9.295649	22.33807	35153.37	310.3759	6.696772	1622.665	3948.991	384.6771	-107.702	64.36137	156.5815	-.0229	-221.587	-163.904	-7.2596
65	-5	5	0	0	9.727915	23.88284	35291.60	257.0511	5.365743	1706.943	3855.053	380.2786	-127.365	80.48751	163.4159	-.0642	-2251.79	-1202.36	-225.069
69	-3	5	0	0	9.554792	23.25757	35227.72	278.8529	5.876309	1673.381	3893.481	382.0359	-119.585	73.29845	160.7377	.0208	453.6582	286.8058	15.7194
71	-2	5	0	0	9.468127	22.94743	35198.00	289.4789	6.144636	1656.424	3912.267	382.9004	-115.624	70.47653	159.4664	.0190	328.9987	219.8248	10.0185
73	-1	5	0	0	9.381575	22.63994	35171.45	299.9779	6.418166	1639.446	3930.790	383.7635	-111.651	67.70195	158.1449	-.0076	-102.216	-71.7318	-1.165
74	-.5	5	0	0	9.338441	22.48790	35160.88	305.1883	6.556851	1631.002	3939.941	384.2096	-109.670	66.13951	157.4075	-.0199	-228.989	-164.839	-6.4703
75	+.5	5	0	0	9.253052	22.19059	35149.78	315.5926	6.839650	1614.465	3957.985	385.1787	-105.741	62.33253	155.6406	-.0138	-109.101	-83.0576	-1.883
76	+1	5	0	0	9.210977	22.04586	35151.44	320.7623	6.983365	1606.403	3966.842	385.7243	-103.805	60.06996	154.5784	.0071	44.6832	35.03519	.3661
78	+2	5	0	0	9.128700	21.76713	35175.38	331.0220	7.277182	1590.891	3984.138	387.0058	-100.014	54.95857	152.0425	.0079	24.2133	20.10711	.149
80	+3	5	0	0	9.049422	21.50627	35233.98	341.3500	7.583156	1576.521	4000.949	388.6226	-96.3407	49.49641	148.9779	No Soln.	-1322.51	-1153.39	-61.794
84	+5	5	0	0	8.901548	21.04675	35485.51	362.2977	8.226618	1551.906	4033.014	393.1921	-89.4763	39.37046	141.7177	No Soln.	-10300.3	-9578.81	-2493.32
30	0	3	0	0	9.360371	22.41886	35220.72	284.2082	5.942392	1618.252	3922.509	383.8004	-113.515	70.37394	159.8504	-.0019	-32.3313	-21.6768	-.00979
28	0	4	0	0	9.327772	22.37755	35182.24	297.3696	6.320457	1620.434	3935.845	384.2067	-110.582	67.71812	158.3482	-.0290	-383.240	-268.955	-16.346
25	0	6	0	0	9.264662	22.30388	35138.53	323.2648	7.072634	1625.206	3961.883	385.2754	-104.902	60.30425	154.4793	.0265	168.3500	131.5876	5.0995
23	0	7	0	0	9.235262	22.27735	35142.05	336.0724	7.448705	1628.220	3974.487	386.0417	-102.203	65.80307	152.0366	.0537	187.6290	154.4500	8.4548

Figure 4-10

Terms dropped: $\Delta x \Delta t, \Delta z \Delta t, \Delta t^2$

Terms added: $\Delta x^3, \Delta x^2 \Delta z, \Delta \dot{z}^3, \Delta x \Delta \dot{z}^2, \Delta z \Delta \dot{z}^2, \Delta \dot{z}^4$

The improvement in accuracy brought about by changing the form of the polynomial expansion is clearly seen from the fact that the standard deviation of $\dot{x}_r$ from the least squares calculation has been reduced from .0694 miles/min (rerun 6) to .0249 miles/min. purely as a result of the change in the form of the expansion.

We summarize below the guidance errors versus deviations in mass flow rate and specific impulse:

Rerun No. 10 - Mass Flow Rate Deviations from Nominal

Run No.	Mass Rate Dev. (%)	Dist. of Closest App. Unfocused (Mi)
65	-5	-1,202.4
69	-3	286.8
71	-2	219 8
73	-1	-71.7
74	-.5	-164.8
15 (Nom.)	0	-163.9
75	+.5	-83.1
76	+1	35.0
78	+2	20.1
80	+3	-1,153.4
84	+5	-9,578.8

Rerun No. 10 - Specific Impulse Deviations from Nominal

(Note: Nominal I_{sp} is 5% above that of Figure 4-2)

Run No.	Sp. Impulse Dev. (%)	Dist. of Closest App. Unfocused (Mi)
30	-2	-21.7
28	-1	-269.0
15 (Nom.)	0	-163.9
25	1	131.6
23	2	154.5

It will be recalled that the last rerun based on a low velocity nominal trajectory (rerun no. 6) showed rather poor performance when confronted with off-nominal specific impulse. We see from the above data that this situation has been greatly improved by changing the form of the polynomial expansion.

Rerun No. 9

Summary Chart: Figure 4-12 (1 sheet)

Nominal Trajectory: High Velocity Coplanar

Terms in Polynomial Expansion for $\dot{X}_r$: $\Delta X, \Delta Z, \Delta \dot{Z}, \Delta t,$
$\Delta X^2, \Delta X \Delta Z, \Delta X \Delta \dot{Z}, \Delta Z^2, \Delta Z \Delta \dot{Z}, \Delta \dot{Z}^2, \Delta \dot{Z} \Delta t,$
$\Delta X^3, \Delta X^2 \Delta Z, \Delta \dot{Z}^3, \Delta X \Delta \dot{Z}^2, \Delta Z \Delta \dot{Z}^2, \Delta \dot{Z}^4.$

Method of Determining Constants: Least Squares

Input Data for Least Squares Calculation: Same as reruns 6, 7, 10 (i.e., based on points taken from trajectories simulated in rerun no. 4)

Standard Deviation of $\dot{X}_r$ from Least Squares Calculation: .0345 mi./min.

As in rerun no. 10, this rerun also used the new form for the polynomial expansion. This rerun, however, is based on the high velocity nominal trajectory. Comparing results of this rerun with rerun no. 7 (also high velocity nominal trajectory, but polynomial expansion in the original form), we see that the change in form of the expansion has reduced the standard deviation of $\dot{X}_r$ from the least squares calculation from .1047 mi./min. to .0345 mi./min.

We summarize below the guidance errors versus deviations in mass flow rate and specific impulse:

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 9

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
21	Nominal Trajectory 0	8	0	0	9.308093	22.79505	36136.80	354.4259	8.018085	1671.176	3976.533	397.0638	-104.738	30.80596	134.8012	-.0168	32.60779	31.37442	1.574
100	-5	8	0	0	9.830130	24.85684	37108.97	307.5857	7.005994	1793.470	3871.333	401.3916	-129.265	25.31707	130.2992	-1.3786	2414.782	2357.855	2159.186
104	-3	8	0	0	9.629912	24.07024	36807.09	327.3856	7.389697	1748.229	3913.668	400.6619	-120.056	26.57039	131.1885	.0110	-20.6994	-20.1523	-.87253
106	-2	8	0	0	9.523291	23.64533	36590.42	336.6305	7.587402	1722.865	3934.994	399.5768	-115.014	27.72734	132.2261	.0608	-112.989	-109.680	-22.4718
108	-1	8	0	0	9.415446	23.21695	36361.42	345.6053	7.798203	1696.935	3956.013	398.3263	-109.857	29.15811	133.4661	.0013	-3.16266	-3.05807	-.0162
109	-.5	8	0	0	9.361700	23.00516	36248.22	350.0326	7.907090	1684.036	3966.334	397.6930	-107.291	29.95436	134.1237	-.0170	32.38888	31.24503	1.5843
110	+.5	8	0	0	9.254476	22.58629	36025.86	358.8042	8.132091	1658.358	3986.638	396.4267	-102.184	31.73287	135.5117	-.0010	2.578563	2.47365	.0095
111	1	8	0	0	9.200890	22.37810	35914.67	363.1035	8.247535	1645.498	3996.613	395.7723	-99.6356	32.75888	136.2730	.0213	-40.2026	-38.4330	-1.94783
113	2	8	0	0	9.092348	21.95649	35678.20	371.4142	8.485226	1619.152	4016.321	394.2793	-94.4489	35.39003	138.1390	.0238	-42.6359	-41.3060	-1.86025
115	3	8	0	0	8.978272	21.50928	35386.78	379.2012	8.729884	1590.610	4036.081	392.1615	-88.9486	40.02500	141.1757	-.1016	138.2227	128.1219	13.1403
116	3.5	8	0	0	8.916758	21.26388	35196.99	382.7233	8.850835	1574.586	4046.139	390.6159	-85.9757	44.60083	143.8254	No Soln.	-319.428	-288.422	-41.0352
117	4	8	0	0	8.849208	20.98869	34949.33	385.8420	8.968066	1556.251	4056.567	388.4430	-82.7122	55.11618	148.6833	No Soln.	-5303.19	-4397.21	-1884.52
25	0	6	0	0	9.405723	23.06584	36572.90	330.4846	7.368546	1680.571	3946.563	400.1011	-112.442	28.01972	132.9875	.0108	-20.0329	-19.4549	-.71932
23	0	7	0	0	9.357694	22.93434	36352.63	342.6216	7.694787	1676.239	3961.587	398.5711	-108.585	29.32466	133.8482	-.0171	32.07286	30.97274	1.63626
19	0	9	0	0	9.257319	22.65020	35922.19	365.9095	8.339816	1665.540	3991.343	395.5405	-100.898	32.54667	135.8938	.0013	-2.44854	-2.3697	-.0075
17	0	10	0	0	9.204596	22.49573	35700.75	377.0226	8.659162	1659.026	4006.806	393.9180	-97.0221	34.7513	137.2576	.0058	-10.3013	-9.80688	-.10978

Figure 4-12

Rerun No. 9 - Mass Flow Rate Deviations from Nominal

Run No.	Mass Rate Dev. (%)	Dist. of Closest App. Unfocused (Mi)
100	-5	2,357.9
104	-3	-20.2
106	-2	-109.7
108	-1	-3.1
109	-.5	31.2
21 (Nom.)	0	31.4
110	+.5	2.5
111	+1	-38.4
113	+2	-41.3
115	+3	128.1
116	+3.5	-288.4
117	+4	-4,397.2

Rerun No. 9 - Specific Impulse Deviations from Nominal

(Note: Nominal I_{sp} is 8% above that of Figure 4-2)

Run No.	Sp. Impulse Dev. (%)	Dist. of Closest App. Unfocused (Mi)
25	-2	-19.5
23	-1	31.0
21 (Nom.)	0	31.4
19	+1	-2.4
17	+2	-9.8

It can be seen that using the high velocity nominal trajectory has greatly improved performance over the low velocity results of rerun no. 10, particularly with respect to specific impulse variations. Fundamentally this improvement is explained by the reduction of sensitivity of miss to velocity with increase in velocity.

Rerun No. 14

Summary Chart: Figure 4-13 (3 sheets)

Nominal Trajectory: Low Velocity Coplanar

Terms in Polynomial Expansion for $\dot{X}_r$: $\Delta X, \Delta Z, \Delta \dot{Z}, \Delta t, \Delta X^2, \Delta X \Delta Z, \Delta X \Delta \dot{Z}, \Delta Z^2, \Delta Z \Delta \dot{Z}, \Delta \dot{Z}^2, \Delta \dot{Z} \Delta t, \Delta X^3, \Delta X^2 \Delta Z, \Delta \dot{Z}^3, \Delta X \Delta \dot{Z}^2, \Delta Z \Delta \dot{Z}^2, \Delta \dot{Z}^4$.

Method of Determining Constants: Least Squares

Input Data for Least Squares Calculation: Up to this point all reruns involving the least squares technique used a collection of input points for the least squares analysis which were selected in the same manner. Specifically, the points taken had the same deviation from nominal injection as the cutoff points from rerun no. 4, plus a point shortly before and shortly after cutoff.

In this and the next rerun (no. 13), a different method was used for selecting the input data. From the results of previous reruns it was noted that the variables at injection were functionally related. This can be made clearer as follows: Imagine the independent injection conditions to be plotted in a multidimensional space with coordinate axes X, Z, $\dot{Z}$, t. One might expect these points to form a random cloud. However, this is not the case. It is found that the injection points lie along fairly well defined curves in this multidimensional space. Injection points resulting from trajectories with off nominal mass flow rates generate one such curve. Off nominal specific impulse generates another curve. In this and the next rerun, an attempt was made to take advantage of this systematic relation between injection points from different trajectories. This was based on the feeling that the polynomial expansion for $\dot{X}_r$ must be very accurate only at injection. Therefore, the points used in the least squares analysis should be typical of injection conditions. Therefore, the following procedure was followed: Based on data from previous reruns using the same nominal trajectory, a set of curves was drawn of injection values of X, Z, $\dot{Z}$, and t versus mass flow deviation, and another set of the same variables versus specific impulse deviation. Sets of variables (X, Z, $\dot{Z}$, t) for the least squares computation were then obtained by reading these curves at many values of mass flow and specific impulse deviations.

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 14

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI.)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
15	Nominal Trajectory																		
	0	+5	0	0	9.295720	22.33844	35154.04	310.3784	6.696882	1622.690	3948.983	384.6840	-107.706	64.27856	156.5555	-.0144	-138.678	-102.738	-2.68530
23	0	7	0	0	9.234852	22.27519	35138.19	336.0504	7.447937	1628.062	3974.527	386.0018	-102.182	56.07883	152.1475	.0027	9.612636	7.865643	.0217438
24	0	6.5	0	0	9.249443	22.28798	35135.01	329.6616	7.260047	1626.526	3968.253	385.6065	-103.524	58.31398	153.3835	.0191	94.39013	75.56194	1.824219
25	0	6	0	0	9.264498	22.30302	35136.96	323.2565	7.072309	1625.137	3961.900	385.2592	-104.894	60.45337	154.5318	.0066	42.02864	32.76775	.3150177
26	0	5.5	0	0	9.279950	22.31996	35143.49	316.8308	6.884596	1623.868	3955.474	384.9536	-106.289	62.45401	155.5892	-.0079	-63.1302	-48.0103	-.626754
27	0	4.5	0	0	9.311775	22.35824	35167.93	303.8962	6.509077	1621.590	3942.427	384.4431	-109.142	65.91851	157.4389	-.0116	-131.024	-94.6841	-2.15457
28	0	4	0	0	9.327970	22.37860	35184.19	297.3768	6.320838	1620.509	3935.822	384.2207	-110.592	67.42850	158.2648	-.0049	-63.1129	-44.5283	-.453155
29	0	3.5	0	0	9.344226	22.39911	35202.04	290.8162	6.132066	1619.418	3929.176	380.0089	-112.052	68.88049	159.0588	.0004	5.493368	3.744349	.0030517
30	0	3	0	0	9.360315	22.41855	35220.01	284.2044	5.942243	1618.230	3922.513	383.7944	-113.512	70.47503	159.8780	-.0090	-152.215	-101.750	-2.15188
37	0	5	0	+.2	9.291266	22.37944	35160.94	306.0283	6.555195	1623.861	3943.797	384.4140	-108.953	65.67078	157.2699	-.0163	-179.153	-129.954	-4.09219
38	0	5	0	+.15	9.292365	22.36913	35159.03	307.1159	6.590602	1623.565	3945.095	384.4801	-108.641	65.33387	157.0962	-.0164	-174.174	-127.000	-3.95387
39	0	5	0	+.1	9.293475	22.35887	35157.22	308.2030	6.626035	1623.271	3946.391	384.5467	-108.329	64.99269	156.9202	-.0163	-169.029	-123.892	-3.80757
40	0	5	0	+.05	9.294597	22.34866	35155.56	309.2916	6.661499	1622.982	3947.688	384.6148	-108.017	64.64042	156.7397	-.0158	-156.735	-115.490	-3.34988
41	0	5	0	-.05	9.296887	22.32846	35152.67	311.4675	6.732512	1622.417	3950.272	384.7546	-107.396	63.90749	156.3667	-.0127	-117.211	-87.3113	-1.96540
42	0	5	0	-.1	9.298055	22.31847	35151.48	312.5564	6.768088	1622.142	3951.562	384.8268	-107.086	63.52477	156.1731	-.0100	-88.9749	-66.6485	-1.16122
43	0	5	0	-.15	9.299229	22.30848	35150.47	313.6419	6.803482	1621.865	3952.850	384.9005	-106.778	63.13182	155.9754	-.0064	-54.8265	-41.3096	-.452545
44	0	5	0	-.2	9.300425	22.29864	35149.59	314.7352	6.839214	1621.601	3954.139	384.9756	-106.469	62.73201	155.7733	-.0027	-22.2173	-16.8528	-7.64312

Figure 4-13
(No. 1 of 3 Sheets)

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 14 (Cont'd)

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI.)	Z (MI.)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS.)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
65	-5	5	0	0	9.728323	23.88505	35295.30	257.0686	5.366874	1707.098	3855.003	380.3160	-127.386	79.32973	163.1692	-.0184	-617.979	-340.579	-19.1597
67	-4	5	0	0	9.650325	23.57919	35263.31	264.1456	5.509454	1689.327	3870.547	380.9559	-124.292	79.92186	163.0014	-.1230	-3946.53	-2159.00	-694.161
69	-3	5	0	0	9.554473	23.25589	35224.77	278.8364	5.875520	1673.260	3893.514	382.0058	-119.569	73.93168	160.8944	-.0157	-351.167	-219.239	-9.05697
71	-2	5	0	0	9.467906	22.94628	35195.93	289.4731	6.144249	1656.343	3912.295	382.8795	-115.612	70.84308	159.5637	-.0063	-109.705	-72.8424	-1.09075
73	-1	5	0	0	9.381541	22.63973	35171.10	299.9772	6.418081	1639.431	3930.795	383.7600	-111.649	67.75272	158.1594	-.0119	-158.519	-111.190	-2.79586
76	+1	5	0	0	9.211121	22.04662	35152.84	320.7691	6.983642	1606.458	3966.826	385.7388	-103.812	59.93903	154.5318	.0252	153.5883	120.5933	4.346603
78	+2	5	0	0	9.129211	21.76982	35180.49	331.0474	7.278091	1591.087	3984.087	387.0589	-100.040	54.62076	151.9017	.0766	222.3944	185.2912	12.79875
79	+2.5	5	0	0	9.089724	21.64017	35210.96	336.2565	7.431356	1583.989	3992.518	387.8837	-98.2131	51.62139	150.2891	-.1288	-167.351	-143.403	-8.83416
80	+3	5	0	0	9.051620	21.51788	35256.51	341.4660	7.586906	1577.374	4000.737	388.8581	-96.4558	48.49607	143.4725	No Soln.	-1088.99	-956.660	-386.929
160	0	5	+.25	0	9.295618	22.38914	35152.91	310.3735	6.696753	1626.182	3947.541	384.5759	-108.044	64.41648	156.5986	-.0075	-72.8850	-53.8996	-.735580
161	0	5	+.5	0	9.295464	22.43954	35151.41	310.3664	6.696451	1629.653	3946.102	384.4637	-108.380	64.60083	156.6563	-.0054	-54.0005	-39.8337	-.399139
162	0	5	+.75	0	9.295263	22.48971	35149.46	310.3571	6.696041	1633.103	3944.665	384.3466	-108.713	64.8435	156.7319	-.0094	-95.4301	-70.1180	-1.22625
132	0	5	+1	0	9.294579	22.53730	35142.83	310.3256	6.694625	1636.368	3943.278	384.1309	-109.021	65.69910	156.9943	-.0722	-772.803	-560.348	-74.9478
142	-2.5	4	+.5	-.1	9.550525	23.24692	35276.34	273.0683	5.710511	1670.374	3888.476	382.2374	-120.786	68.93037	159.9386	.3732	7084.984	4844.120	3016.571
143	-2.5	4	+.5	+.1	9.545780	23.28781	35282.26	268.6285	5.569611	1671.392	3883.206	381.9164	-122.018	71.09951	160.8074	.2727	5908.477	3879.656	2103.316
144	-2.5	6	+.5	+.1	9.471043	23.15476	35142.22	295.0261	6.303915	1672.750	3911.348	382.1966	-115.762	77.36666	160.8177	-.4040	-8958.02	-5261.17	-3173.64
145	+2.5	6	+.5	-.1	9.067498	21.78466	35300.95	347.143	7.752718	1598.091	3998.621	389.1772	-97.2632	45.92099	146.7739	No Soln.	-1158.98	-1037.02	-485.873
146	+2.5	6	+.5	-.1	9.078180	21.77767	35359.95	351.7801	7.906301	1599.324	4003.121	390.0991	-96.3328	43.46271	145.0415	No Soln.	-2778.85	-2523.35	-1883.14

Figure 4-13
(No. 2 of 3 Sheets)

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 14 (Cont'd)

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
147	+2.5	4	+.5	-.1	9.111839	21.69931	35121.35	325.2507	7.106246	1584.039	3980.656	386.2310	-100.555	61.15787	154.7034	-.6421	-3367.44	-2603.95	-1399.94
148	+2.5	4	+.5	+.1	9.105872	21.73069	35109.56	320.9298	6.960464	1584.620	3975.774	385.7886	-101.715	64.35984	156.0408	-.7046	-4996.44	-3700.05	-2247.41
149	-2.5	6	+.5	-.1	9.473923	23.10363	35117.57	299.3956	6.440640	1670.974	3916.857	382.3057	-114.428	79.12031	160.9232	- 5678	-12748.0	-7248.53	-4943.96

Figure 4-13
(No. 3 of 3 Sheets)

Standard Deviation of $\dot{x}_r$ from Least Squares Calculation: .0152 mi/min.

The conditions of this rerun were the same as those of rerun no. 10, except for the abovementioned new method for selecting the input data to the least squares analysis. The improvement thus gained can be seen from the fact that the standard deviation of $\dot{x}_r$ from the least squares calculation was reduced from .0249 mi/min in rerun no. 10 to .0152 mi/min in the present rerun.

We summarize below the guidance errors versus deviations in mass flow rate and specific impulse:

Rerun No. 14 - Mass Flow Rate Deviations from Nominal

Run No.	Mass Rate Dev. (%)	Dist. of Closest App. Unfocused (Mi)
65	-5	-340.6
67	-4	-2,159.0
69	-3	-219.2
71	-2	-72.8
73	-1	-111.2
15 (Nom.)	0	-102.7
76	+1	120.6
78	+2	185 3
79	+2.5	-143.4
80	+3	-956.7

Rerun No. 14 - Specific Impulse Deviations from Nominal

(Note: Nominal I_{sp} is 5% above that of Figure 4-2)

Run No.	Sp. Impulse Dev. (%)	Dist. of Closest App. Unfocused (Mi)
30	-2	-101.8
29	-1.5	3.7
28	-1	-44.5
27	-.5	-94.7
15 (Nom.)	0	-102.7
26	+.5	-48.0
25	+1	32.8
24	+1.5	75.6
23	+2	7.9

This rerun was the first on which the effect of delayed lift-off was investigated. The importance of this behavior in the operational concept for dealing with delayed launch was discussed in Section 3.7. From the data presented (see runs 160, 161, 162, 132) it appears that a delay of about 45 seconds is permissible before changing the launch conditions set into the guidance system, i.e., the azimuth heading and guidance constants.

A very significant aspect of this rerun is that it is the first on which performance was evaluated with several different types of off-nominal deviations applied simultaneously. Examination of these results (runs 142 to 149 inclusive) shows that performance under these conditions is totally unacceptable. These are realistic conditions since, in general, off-nominal deviations will occur in more than one parameter simultaneously. This problem was solved during the non-coplanar investigation and will be discussed more fully in Section 4.4.3.

Rerun No. 13

Summary Chart: Figure 4-14 (2 sheets)

Nominal Trajectory: High Velocity Coplanar

Terms in Polynomial Expansion for $\dot{X}_f$: $\Delta X, \Delta Z, \Delta \dot{Z}, \Delta t, \Delta X^2, \Delta X \Delta Z, \Delta X \Delta \dot{Z}, \Delta Z^2, \Delta Z \Delta \dot{Z}, \Delta \dot{Z}^2, \Delta \dot{Z} \Delta t, \Delta X^3, \Delta X^2 \Delta Z, \Delta \dot{Z}^3, \Delta X \Delta \dot{Z}^2, \Delta Z \Delta \dot{Z}^2, \Delta \dot{Z}^4$

Method of Determining Constants: Least Squares

Input Data for Least Squares Calculation: Determined by same method as used for rerun no. 14, i.e , by utilizing the functional dependence between X , Z , $\dot{Z}$, t at injection as determined from trajectories on earlier reruns.

Standard Deviation of $\dot{X}_f$ from Least Squares Calculation 0.0270 mi/min

This rerun is identical with rerun no. 14 except that in this case the high velocity nominal trajectory was used. As in previous high velocity reruns, increased accuracy is obtained because of the decrease in sensitivity of miss to velocity as velocity is increased.

SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

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RERUN NO. 13

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
	Nominal Trajectory																		
21	0	+8	0	0	9.308335	22.79637	36138.77	354.4354	8.018449	1671.240	3976.516	397.0845	-104.747	30.78695	134.7809	.0025	-4.67998	-4.52937	-.031284
17	0	+10	0	0	9.204512	22.49527	35700.02	377.0178	8.659024	1658.991	4006.094	393.9105	-97.0179	34.76171	137.2666	-.0012	2.196789	2.051960	.0047989
18	0	+9.5	0	0	9.231035	22.71913	35811.03	371.5031	8.499303	1662.325	3998.741	394.7282	-98.9581	33 59586	136.5532	-.0125	22.87104	21.76959	.5864182
19	0	+9	0	0	9.257200	22.64954	35920.92	365.9030	8.339574	1665.492	3991.356	395.5273	-100.891	32.56117	135.9080	-.0112	20.82055	19.88817	.5275345
20	0	8.5	0	0	9.282913	22.72363	36030.10	360.2120	8.179274	1668.452	398.3949	396.3121	-102.820	31.63115	135.3202	-.0047	8.808139	8.424789	.1014938
23	0	+7	0	0	9.357918	22.93558	36355.27	342.6345	7.695325	1676.328	396.1563	398.5992	-108.597	29.30275	133.8229	.0095	-17.9184	-17.3439	-.515033
24	0	+6.5	0	0	9.382010	23.00165	36463.48	336.6004	7.532233	1678.537	395.4073	399.3469	-110.519	28.64266	133.3955	.0070	-13.0754	-12.6852	-.290733
25	0	+6	0	0	9.405615	23.06522	36571.52	330.4780	7.368258	1680.525	3946.574	400.0866	-112.436	28 02964	132.9997	-.0031	5.731762	5.534502	.0582314
92	0	8	0	+.2	9.312148	22.88352	36227.63	350.4188	7.893892	1675.756	3970.255	397.6694	-106.478	30.12132	134.3246	.0091	-17.1193	-16.5324	-.438644
93	0	8	0	+.15	9.311182	22.86165	36205.45	351.4248	7.925006	1674.632	3971.822	397.5241	-106.045	30.28321	134.4361	.0075	-14.2004	-13.7116	-.298000
94	0	8	0	+.1	9.310215	22.83979	36183.28	352.4297	7.956175	1673.506	3973.387	397.3786	-105.612	30.44765	134.5488	.0065	-12.1917	-11.7695	-.216796
95	0	8	0	+.05	9.309239	22.81789	36161.04	353.4337	7.987333	1672.375	3974.952	397.2320	-105.179	30.61567	134.6639	.0047	-8.75790	-8.45788	-.110527
96	0	8	0	-.05	9.307274	22.77400	36116.45	355.4364	8.049637	1670.106	3978.077	396.9360	-104.314	30.96173	134.9000	0	-.324866	-.324614	-.000156
97	0	8	0	-.1	9.306278	22.75201	36094.05	356.4353	8.080806	1668.966	3979.639	396.7865	-103.881	31.14044	135.0214	-.0027	5.147573	4.920108	.0359268
98	0	8	0	-.15	9.305293	22.73007	36071.72	357.4328	8.111985	1667.827	3981.199	396.6371	-103.449	31.32169	135.1439	-.0048	9.023409	8.639220	.1092453
99	0	8	0	-.2	9.304300	22.70809	36049.33	358.4285	8.143165	1666.683	3982.757	396.4866	-103.017	31.50705	135.2689	-.0074	13.73371	13.15802	.2498398
100	-5	8	0	0	9.843275	24.93087	37242.86	308.2655	7.044291	1798.756	3869.629	402.7589	-129.983	24.63307	129.2888	-.0451	77.92586	76.18791	14.33999
102	-4	8	0	0	9.737454	24.50332	37033.37	317.9401	7.210125	1773.844	3891.746	401.8105	-125.097	25 49995	130.1475	0605	-106.900	-104.355	-24.3714

Figure 4-14
(No. 1 of 2 Sheets)

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SUMMARY OF TRAJECTORY SIMULATIONS FOR 2-DIMENSIONAL LUNAR IMPACT MISSION

RERUN NO. 13 (Cont'd)

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS									BALLISTIC FLIGHT DATA		GUIDANCE EQUATION PERFORMANCE			
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	DOWN RANGE ANGLE (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	X (MI.)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS.)	SWEEP ANGLE (DEG)	ERROR IN $\dot{X}$ (MI/MIN)	MISS ALONG ORBIT UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
104	-3	8	0	0	9.630144	24 07153	36809.45	327.3979	7.390236	1748.322	3913.639	400.6861	-120.068	26.55585	131.1691	.0349	-63.0848	-61.4402	-7.99906
106	-2	8	0	0	9.522551	23.64128	36582.99	336.5891	7.585749	1722.570	3935.077	399.5005	-114.974	27.77951	132.2914	-.0125	22.97564	22.27632	.9615097
108	-1	8	0	0	9.415178	23.21551	36358.79	345.5905	7.797692	1696.829	3956.041	398.2992	-109.843	29.17958	133.4910	-.0243	45.28110	43.75625	3.294120
111	+1	8	0	0	9.201056	22.37899	35916.32	363.1129	8.247841	1645.564	3996.596	395.7895	-99.6441	32.73951	136.2541	.0381	-70.5857	-67.5096	-5.95892
113	+2	8	0	0	9.092302	21.95625	35677.78	371.4111	8.485138	1619.133	4016.325	394.2749	-94.4467	35.39624	138.1443	.0203	-35.6056	-33.7352	-1.24127
114	+2.5	8	0	0	9.036374	21.73824	35543.39	375.4259	8.607928	1605.327	4026.185	393.3425	-91.7530	37.27926	139.4144	-.0238	38.46781	36.09201	1.252616
115	+3	8	0	0	8.978822	21.51218	35392.24	379.2348	8.730692	1590.825	4036.032	392.2189	-88.9765	39.90336	141.0900	-.0343	46.54749	43.13522	1.514648
163	0	8	+ 25	0	9.309121	22.85185	36147.47	354.4851	8.020215	1675.140	3974.928	397.0810	-105.147	30.70318	134.6911	-.0193	36.30390	34.91461	1.865566
164	0	8	+.5	0	9.310151	22.90879	36157.98	354.5608	8.022735	1679.118	3973.331	397.0962	-105.555	30.60248	134.5819	-.0218	39.59787	38.09856	2.237300
165	0	8	+.75	0	9.311493	22.96712	36171.25	354.6212	8.024977	1683.187	3971.675	397.1384	-105.983	30.47798	134.4480	0	-.107426	-.067177	-.000004
137	0	8	+1	0	9.313047	23.02675	36186.89	354.7105	8.028119	1687.355	3970.004	397.2055	-106.420	30.33262	134.2896	.0467	-88.2078	-85.0189	-11.2212
152	-2.5	+7	+.5	-.1	9.632151	24.09976	36959.79	322.0585	7.252932	1748.070	3907.904	401.9725	-121.721	25.81118	130.4699	.6566	-1173.41	-1143.94	-947.596
153	-2.5	+7	+.5	+.1	9.636040	24.19090	37052.76	317.8574	7.134411	1752.563	3901.286	402.5344	-123.501	25.39954	130.1465	.6766	-1198.54	-1169.54	-979.220
154	-2.5	9	+.5	+.1	9.539623	23.92268	36642.21	342.7799	7.765391	1744.386	3932.228	399.9420	-115.870	27.22028	131.2737	.9741	-1785.04	-1733.71	-1506.24
155	+2.5	9	+.5	+.1	8.981000	21.70584	35312.17	383.9117	8.867795	1606.188	4034.977	391.2437	-89.1616	41.30481	141.7612	-.3765	447.3398	411.9387	116.6683
156	+2.5	9	+.5	-.1	8.974269	21.60704	35199.45	387.5396	8 994629	1600.564	4041.113	390.3418	-87.3404	43.93519	143.1482	.3380	-208.323	-188.939	-22.6311
157	+2.5	7	+.5	-.1	9.082885	21.92945	35699.34	366.1795	8.335825	1615.300	4012.228	394.3103	-95.3471	35.38662	138.4303	-.59029	1048.286	992.8422	634.7816
158	+2.5	7	+.5	+.1	9.088547	22.02379	35806.29	362.4205	8 210293	1620.495	4006.078	395.1216	-97.1498	34.11690	137.5825	-.4468	816.1516	776.6973	463.0078
159	-2.5	9	+.5	-.1	9.536188	23.83479	36557.59	346.9623	7.887504	1740.042	3938.726	399.4395	-114.139	27.67223	131.5871	.0207	-1880.23	-1823.95	-1587.44

Figure 4-14
(No. 2 of 2 Sheets)

We summarize below the guidance errors versus deviations in mass flow rate and specific impulse.

Rerun No. 13 - Mass Flow Rate Deviations from Nominal

Run No.	Mass Rate Dev. (%)	Dist. of Closest App. Unfocused (Mi)
104	-3	-61.4
106	-2	22.3
108	-1	43.8
21 (Nom.)	0	-4.5
111	+1	-67.5
113	+2	-33.7
114	+2.5	36.1
115	+3	43.1

Rerun No. 13 - Specific Impulse Deviations from Nominal

(Note: Nominal I_{sp} is 8% above that of Figure 4-2)

Run No.	Sp. Impulse Dev. (%)	Dist. of Closest App. Unfocused (Mi)
25	-2	5.5
24	-1.5	-12.7
23	-1	-17.3
21 (Nom.)	0	-4.5
20	+.5	8.4
19	+1	19.9
18	+1.5	21.8
17	+2	2.1

Let us review briefly the differences between the first rerun utilizing least squares (rerun no 6) and the present rerun·

....change in mathematical form of expression

....different method of selecting input data for least squares calculation

....higher velocity nominal trajectory

The combined effect of these 3 changes is to bring about a tremendous improvement in system performance when confronted with singly applied off nominal deviations in mass flow rate or specific impulse. This improvement is shown below:

Mass Flow Dev. (%)	Miss (Rerun No. 6)	Miss (Rerun No. 13)
-3	328.2	-61.4
+3	-1,377.9	43.1

Sp Impulse Dev. (%)	Miss (Rerun No. 6)	Miss (Rerun No. 13)
-2	-1,240.4	5.5
+2	81.1	2.1

In the case of different types of off-nominal deviations applied simultaneously (see runs 152 to 159 inclusive), the use of the high velocity nominal trajectory causes errors to be somewhat smaller than were obtained in rerun 14 using the low velocity nominal trajectory. In spite of this improvement, however, the errors are still too large to be considered acceptable.

Conclusions from Coplanar Trajectory Simulations

We have concluded our presentation of the coplanar trajectory simulations carried out during this study. Before proceeding to a discussion of the more realistic non-coplanar investigation, let us summarize the main conclusions of the coplanar study.

Prior to the present study, it was widely thought that when an inertial guidance system is used for lunar missions, the prime source of error would be the inertial components, any contribution due to the guidance concept being completely negligible. However, the work presented thus far shows that it is not a trivial matter to derive guidance equations which contribute negligible errors. We shall have more to say on this question later in this volume.

A more specific conclusion drawn from the coplanar results concerns the manner in which the input conditions are selected for the least squares analysis leading to the constants of the polynomial expansion. In the

coplanar studies, an attempt was made to take advantage of the functional relation between the navigation variables at injection. This was found to work quite well as long as the off-nominal deviations were applied one at a time. However, when off-nominal deviations of different types are applied simultaneously, the guidance accuracy deteriorates considerably. Hence, it is necessary to choose the inputs for the least squares computation of constants which does not place such heavy emphasis on the functional relationship between the navigation variables at injection taken from trajectories which included only a single off-nominal deviation. The method derived for accomplishing this will be discussed in connection with the non-coplanar investigation.

4.4 3 Results of Non-Coplanar Trajectory Simulations

4.4.3.1 General Remarks

Because of the coplanar nature of the data presented previously, u and $\dot{y}$ were always zero. Hence, the only dependent variable computed by the polynomial expansion was $\dot{x}_r$, and the only independent variables entering the polynomial expansion were x, z, $\dot{z}$, t.

However, in the non-coplanar runs to be presented next, both $\dot{x}_r$ and $\dot{y}_r$ are dependent variables which must be computed by polynomial expansions, and the independent variables entering the polynomial expansions for $\dot{x}_r$ and $\dot{y}_r$ will now be the complete set x, y, z, $\dot{z}$, t.

In addition, in the non-coplanar case, the guidance system must not only control engine cut-off, but must also control the steering of the missile in azimuth.

Although the technical approach to the guidance system formulation was discussed generally in Section 3, it is appropriate to consider some details of the system which were left undefined in Section 3 before proceeding to examine the simulation results.

4.4.3.2 Non-Coplanar Polynomial Expansions

The polynomial expansion for $\dot{x}_r$ included the following terms:

....linear $\Delta x, \Delta y, \Delta z, \Delta \dot{z}, \Delta t$

....second order $\Delta x^2, \Delta x \Delta y, \Delta x \Delta z, \Delta x \Delta \dot{z}$
$\Delta y^2, \Delta y \Delta z, \Delta y \Delta \dot{z}$
$\Delta z^2, \Delta z \Delta \dot{z}$
$\Delta \dot{z}^2, \Delta \dot{z} \Delta t$

....third order $\Delta x^3, \Delta x^2 \Delta z$
$\Delta \dot{z}^3, \Delta \dot{z}^2 \Delta x, \Delta \dot{z}^2 \Delta y, \Delta \dot{z}^2 \Delta z$

....fourth order $\Delta \dot{z}^4$

This includes all the terms in the later coplanar results (reruns no. 10, 9, 14, 13) as well as the following new terms in Δy: $\Delta y, \Delta y^2, \Delta y \Delta x, \Delta y \Delta z, \Delta y \Delta \dot{z}, \Delta y \Delta \dot{z}^2$

Because of the short time available for the study, it was not possible to conduct a detailed investigation to optimize the number of terms in the expansion for $\dot{y}_r$. Hence the expansion for $\dot{y}_r$ was taken as having the same terms as those given above for $\dot{x}_r$ (although obviously the constant coefficients were separately determined for each expansion).

It will be recalled that in the coplanar investigation, a serious difficulty was encountered when the input data to the least squares computation of constants was selected in a manner which utilized the functional relation between the navigation variables at injection. This functional relation was observed in sets of trajectories which were generated with only one type of off nominal deviation applied at a time. In an effort to obtain better performance in the important practical case of several simultaneous off-nominal deviations of different types applied simultaneously, the following method of selecting input data for the least squares calculation was developed during the non-coplanar investigation:

The expected maximum and minimum values at injection of each of the 5 independent variables were determined. For X, Z, $\dot{Z}$, t this data was obtained from the coplanar results. For Y, it was necessary to make some rough calculations since non-coplanar simulations had not yet been run.

The results of this determination of the "spread" of the variables at injection are as follows:

Quantity	Minimum	Nominal	Maximum
X (mi)	1,585	1,684.1	1,810
Y (mi)	-110	-22.6	70
Z (mi)	-89	15.7	91
$\dot{Z}$ (mi/min)	-130	-105.7	-85
t (min)	9.027	9.327	9.927

The nominal values have been included above for information. Next a table of 5 digit random numbers was used to generate sets of off-nominal injection conditions as inputs to the least squares calculation. Each of the five digits was associated with one of the five above variables as follows

First Digit: X

Second Digit: Y

Third Digit: Z

Fourth Digit: $\dot{Z}$

Fifth Digit: t

The value of the individual digits was used to determine at what point in the allowable range the value of the corresponding variable should be fixed. An example will make the procedure clear. A typical random number might be 37842. The corresponding values of X, Y, Z, $\dot{Z}$, t, are computed somewhat as follows:

$$X = 1585 + 0.3\,[1810 - 1585]$$

$$Y = -110 + 0\ 7\,[70 - (-110)]$$

$$Z = -89 + 0.8\,[91 - (-89)]$$

$$\dot{Z} = -130 + 0.4\,[-85 - (-130)]$$

$$t = 9.027 + 0.2\,[9.927 - 9.027]$$

One hundred points were generated in this manner and used as the basis for all least squares analysis. It has been explained previously that, in a supplementary calculation, the least squares analysis program computes the standard deviation of the error between the quantity computed by the expansion and the analytically correct value. In the tables given below, we present this standard deviation data. The misses are calculated using sensitivities computed for the nominal trajectory. Since the nominal trajectory has a fairly high injection velocity, the variation in sensitivity (noted in the discussion of coplanar rerun no. 6) which is so pronounced for low velocity trajectories can be ignored, and hence a single set of sensitivities gives fairly accurate results. The nominal sensitivities of miss to errors in $\dot{X}$ and $\dot{Y}$ are:

$$\frac{\partial \epsilon_V}{\partial \dot{X}} = -1963.9 \text{ mi/mi/min} \qquad \frac{\partial \epsilon_V}{\partial \dot{Y}} = 6.87 \text{ mi/mi/min}$$

$$\frac{\partial \epsilon_H}{\partial \dot{X}} = 583.35 \text{ mi/mi/min} \qquad \frac{\partial \epsilon_H}{\partial \dot{Y}} = 430.28 \text{ mi/mi/min}$$

The miss components ϵ_V and ϵ_H have been defined in Section 3.3. In the following tables, 3 expansions are referred to. These are:

....24 term: This is the expansion which was actually used in the powered flight simulations to be described later. The terms were given earlier in this section.

....12 term: This is an expansion which has been found in other studies to be well suited to terrestrial applications, and it was, therefore, of interest to determine its applicability to space missions. The terms included are: $\Delta x, \Delta y, \Delta z, \Delta \dot{z}, \Delta t, \Delta x^2, \Delta z^2, \Delta \dot{z}^2, \Delta x \Delta z, \Delta x \Delta \dot{z}, \Delta z \Delta \dot{z},$ constant term

....6 term: This is an expansion using only linear terms which was included in order to assess the applicability of rather crude guidance computations. The terms included are: Δx, Δy, Δz, $\Delta \dot{z}$, Δt, constant term

Accuracy of $\dot{x}_r$ Polynomial Expansion
(All entries are 1-sigma values)

Expansion	Error in $\dot{x}_r$ (mi/min)	ϵ_V (mi)	ϵ_H (mi)
24 term	0.06	118	35
12 term	0.25	491	146
6 term	0.40	786	234

Accuracy of $\dot{y}_r$ Polynomial Expansion
(All entries are 1-sigma values)

Expansion	Error in $\dot{y}_r$ (mi/min)	ϵ_V (mi)	ϵ_H (mi)
24 term	0 30	2.1	130
12 term	1.45	10.0	628
6 term	3.35	23.0	1450

The above tables give a rather optimistic evaluation for all the expansions for the following reasons:

....the one-sigma errors in $\dot{x}_r$ and $\dot{y}_r$ were computed only on the basis of the points used to derive the constants in the expansions

....the sensitivities of miss to errors in $\dot{x}$ and $\dot{y}$ are rather low because of the high velocity nominal trajectory.

. . . it was assumed that the only error in $\dot{y}$ at injection is because of the approximation error of the $\dot{y}_r$ polynominal expansion. This implies perfect steering in azimuth in response to commands generated by the guidance system. Actually, as will be seen when the results of the non-coplanar simulations are discussed, there is an additional error in $\dot{y}$ due to imperfect azimuth steering.

It can be seen from the above tabulations that neither of the simpler guidance expansions (12 term or 6 term) yields satisfactory accuracy, particularly when the optimistic nature of the tabulated results is realized.

It is of interest to note that the non-coplanar results with the 24 term expansion actually used in the trajectory simulations yielded larger standard deviations based on the input data than did the coplanar results. For example:

Std. Dev. of Non-Coplanar 24 term expansion for $\dot{x}_r$:
0 06 mi/min

Std. Dev. of Coplanar Rerun No. 13 expansion for $\dot{x}_r$:
0.027 mi/min

This, however, is an apparent, rather than an actual deterioration in performance which can be explained as follows: These standard deviations only indicate the goodness-of-fit of the polynomial expansion to the input data used in the deterioration of the constant coefficients. In the coplanar case, the input variables were more or less functionally related, while in the non-coplanar case they were randomly distributed. Hence, it is clear that the functionally related input variables of the coplanar case will demonstrate better goodness-of-fit than the randomly distributed non-coplanar input variables. This deterioration in standard deviation is the price one pays for vastly improved performance when the system is confronted with more than one type of off-nominal deviation applied simultaneously.

4.4.3 3 Azimuth Steering

The manner in which the guidance computer output is used to control steering in azimuth was discussed in Sections 3.5 and 3.8. However, certain details which were left undefined must be clarified before discussing the results of the non-coplanar trajectory simulations. In this connection, reference to Figure 3-6 will be found useful.

The azimuth guidance scheme used in this study for the lunar impact mission is a direct adaptation of a scheme which other Arma investigations have shown to be admirably well suited to terrestrial applications. It therefore seems natural to investigate the suitability of this azimuth steering scheme to injection guidance for space missions. As will be seen later in this discussion, this azimuth steering scheme, while adequate for hitting the moon under most conditions, cannot be called eminently successful. The Arma Space Research Group has performed cursory studies which have resulted in a number of promising schemes for improving the azimuth steering performance, but the limited time available for this study did not permit detailed exploration of these ideas. This task should be undertaken at the earliest opportunity

To briefly review the main points of the azimuth steering method:

....The $\dot{y}$ velocity to be gained ($\Delta\dot{y}$) is computed by

$$\Delta\dot{y} = \dot{y}_r - \dot{y}$$

where $\dot{y}_r$ is the value of y velocity required for impact as computed by the polynomial expansion, and $\dot{y}$ is the present value of y velocity given by the navigation portion of the system.

....The horizontal error function (HEF) is computed by

$$HEF = \Delta\dot{y} - K\Delta\dot{x}$$

where K is a constant derived by taking the slope of a straight line fitted to a curve of $\Delta\dot{y}$ vs. $\Delta\dot{x}$ plotted for the nominal trajectory.

....The missile azimuth angle relative to the inertial platform (β) is given by

$$\beta = K_{\beta_1} \times HEF$$

...Due to the fact that the abovementioned straight line fit to $\Delta\dot{y}$ vs. $\Delta\dot{x}$ is inaccurate early in the flight, the computed azimuth steering order, HEF, does not start to control azimuth steering until a predetermined switching time, t_{sw}, is reached.

The quantitative values which must be selected before the azimuth steering method can be evaluated are:

....The slope K in the expression

$$HEF = \Delta\dot{y} - K\Delta\dot{x}$$

....The time t_{sw} at which the computed azimuth steering takes control

....The azimuth control system gain constant K_{β_1}

Figure 4-15 shows a plot of $\Delta\dot{y}$ as a function of $\Delta\dot{x}$ for the non-coplanar nominal trajectory (run 210). As explained in Section 3.5, this curve is fitted with a straight line. Here we see one of the difficulties with this azimuth steering scheme,

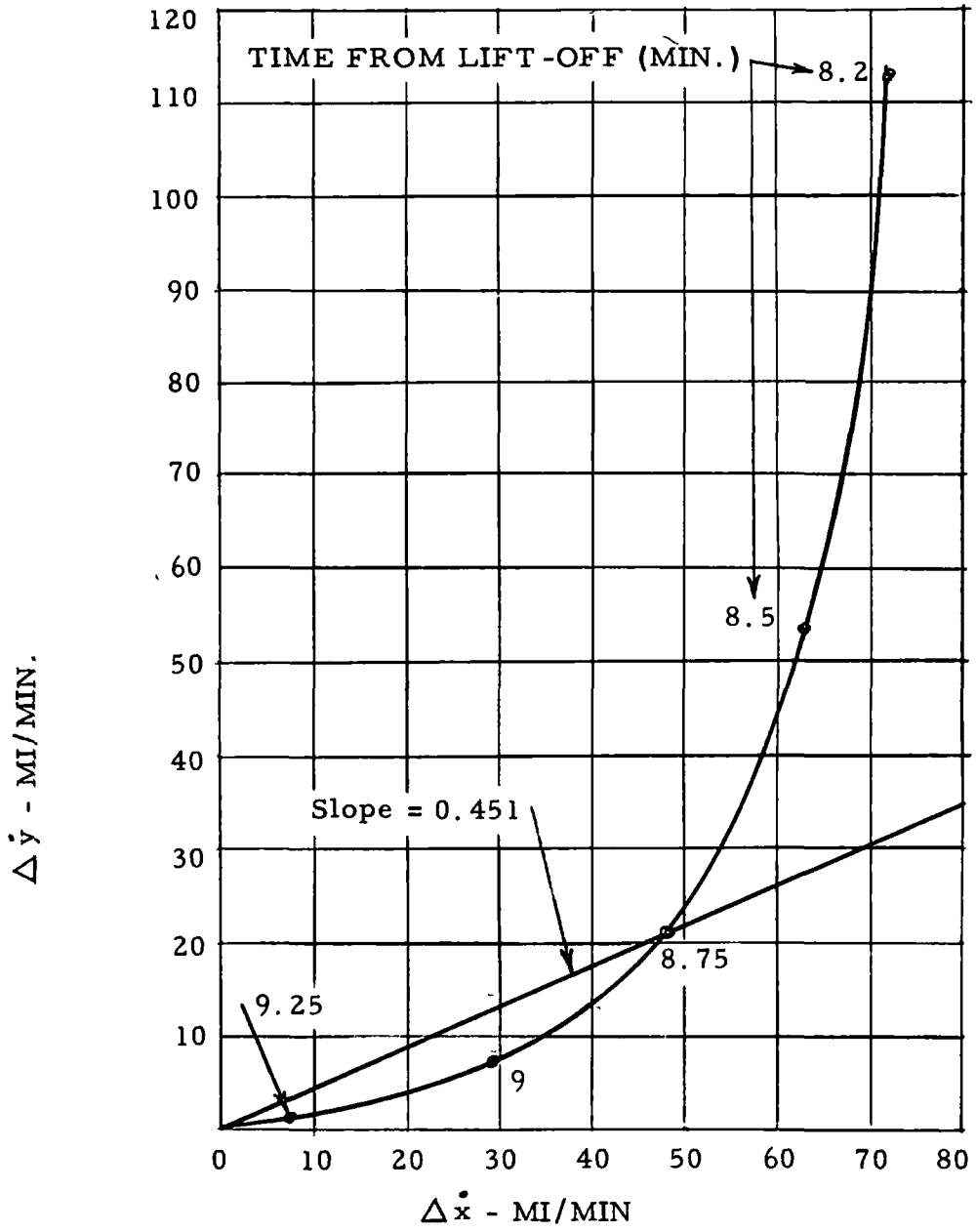

Figure 4-15 - y VELOCITY TO BE GAINED ($\Delta\dot{y}$) VS. x VELOCITY TO BE GAINED ($\Delta\dot{x}$) FOR NON-COPLANAR NOMINAL TRAJECTORY (RUN 210)

i.e., the curvature of this plot is so pronounced that any straight line fits rather poorly. (A promising method of improving this situation would be to fit the curve with a higher order expression, but time did not permit detailed examination of this approach.) As an interim approach (pending more careful study of this problem), a straight line through the origin with slope equal to 0.451 was fitted to the curve By comparing this straight line with the true curve in Figure 4-15, it can be seen why computed azimuth steering must be delayed until quite late in the flight. An erroneous steering signal proportional to the error between the curve and the straight line would be computed even if the missile flew under completely nominal conditions. Hence, it is desirable to wait until the straight line is a fair approximation to the curve before starting computed azimuth steering. On the other hand, there are disadvantages in waiting too long to start the azimuth control. Under certain off-nominal conditions, the powered flight time can be appreciably shorter than nominal. For example, the powered flight time on run no. 261 was 8.963 minutes, as compared with the nominal value of 9 327 minutes. When faced with such a short flight time, a switching time which is too late does not allow enough time for the azimuth steering system to remove errors in $\dot{y}$ before thrust cutoff. As a compromise between the requirements on switching time, a value of t_{sw} equal to 8.35 minutes was selected. (Note. Considerable improvement may be realized by computing the switch time in flight rather than using a predetermined value. For example, the error in $\dot{x}$ is roughly a measure of time to go before cutoff. Hence, switching might be effected when $\Delta\dot{x}$ gets down to a certain value. However, on the present study, there was insufficient time to investigate this approach in detail.)

The value of the azimuth control system gain constant, K_{β_1}, was set equal to 1 rad/mi/min (equivalent to 0.65 deg/ft/sec). In addition a limit of 45 degrees was set for azimuth angle, β. That is if the commanded azimuth $K_{\beta_1} \times$ HEF exceeded 45 degrees, the missile body axis azimuth was held at this limit until the order came inside the region ±45 deg.

4.4.3.4 Summary of Non-Coplanar Simulation Results

The results of the non-coplanar trajectory simulations are shown in Figure 4-16. The quantities tabulated are the following:

Identification

Run No. - This number identifies the off-nominal deviations associated with the run.

Off-Nominal Deviations

Flow Rate - The percentage difference between the flow rate simulated and the flow rate of the standard acceleration profile of Figure 4-2.

Specific Thrust - The percentage difference between the specific impulse simulated and the specific impulse of the standard acceleration profile of Figure 4-2. Note that for the nominal trajectory (run no. 210) this value is 8%.

Lift Off Delay - The time between the start of inertial guidance system operation and the initiation of thrust. The importance of this parameter in the operational concept for dealing with delays in launch was discussed in Section 3.7.

Pitch Offset - A constant error in the preprogrammed pitch profile of Figure 4-3.

Note: A random number system similar to that used in choosing the input injection conditions to the least squares determination of guidance constants (see Section 4.4.3.2) was used to choose the combination of the 4 off-nominal deviations to be used for a particular trajectory. This was to insure a more realistic assignment of off-nominal deviations than was used in the coplanar studies.

Injection Conditions

Time - Time from lift-off to injection. Total system operating time is this quantity plus lift-off delay.

Latitude - Latitude of the vehicle at injection in Earth-fixed coordinates.

Longitude - Change in longitude from start of system operation to injection. Measured positive eastward.

ΔRA- Change in right ascension from start of system operation to injection. Includes change in right ascension of missile on launch pad during delay in lift-off.

Velocity - Magnitude of inertial velocity at injection.

Altitude - Altitude of the vehicle at injection.

Path Angle - The angle of the inertial velocity vector with respect to the local horizontal at injection.

Azimuth Heading - The angle of the horizontal projection of the inertial velocity at injection with respect to North. Measured clockwise.

X , Y , Z - The rectangular components of the vehicle's position at injection measured in the coordinate system of the inertial platform.

$\dot{X}$, $\dot{Y}$, $\dot{Z}$ - The rectangular components of the vehicle's inertial velocity at injection measured in the coordinate system of the inertial platform.

Ballistic Flight Data

Time of Flight - Ballistic flight time from injection to lunar impact, ignoring the effect of lunar gravitation.

Sweep Angle - The geocentric angle swept out by the vehicle between injection and lunar impact, ignoring the effect of lunar gravity.

Impact Velocity - The velocity of the vehicle at lunar impact ignoring the effect of lunar gravity.

Guidance Equation Performance

Error in $\dot{X}$ - The difference between the actual value of $\dot{X}$ at injection and the value of $\dot{X}$ which is required for a perfect lunar impact.

Error in $\dot{Y}$ (total) - The difference between the actual value of $\dot{Y}$ at injection and the value of $\dot{Y}$ which is required for a perfect lunar impact.

Error in $\dot{Y}$ (Azimuth Steering) - Theoretically, the HEF function ($\Delta\dot{Y} - K\Delta\dot{X}$) used to control azimuth steering should be zero at the moment of injection. However, due to the imperfections of the steering scheme, some residual value remains. This is the value listed in this column.

Horizontal Miss - The component of the chordal miss (see below) which is perpendicular to the vehicle's orbital plane.

Vertical Miss - The component of the chordal miss (see below) which lies in the vehicle's orbital plane.

Chordal Miss - The distance between the vehicle and the center of the moon when the vehicle reaches the radius of the moon's orbit, ignoring the effect of lunar gravity.

Distance of Closest Approach Unfocused - The distance of closest approach between the vehicle and the center of the moon ignoring the effect of the lunar gravitational field.

Distance of Closest Approach Focused - The distance of closest approach between the vehicle and the center of the moon including the effect of the lunar gravitational field.

The results of the non-coplanar trajectory simulations are presented in the 3 sheets of Figure 4-16. The first two sheets contain information on trajectories for which the guidance system indicated a cutoff point before the rocket fuel was exhausted. We shall discuss the third sheet later in this section.

There are certain important conclusions which can be drawn from an examination of the trajectory summary data. First of all, the vertical miss is within acceptable limits on all trajectories which yielded cut-off. Almost all of the trajectories involved off-nominal deviations of more than one type applied simultaneously. Thus, we see that the use of the random method for selecting the input data for the least squares calculation of guidance expansion constants has indeed improved performance over the coplanar case. The following summarizes the cutoff accuracy performance for all the non-coplanar trajectories which yielded a cutoff point: (The miss components were computed using the nominal sensitivities given in Section 4.4.3.2.)

RMS error in $\dot{X}$: 0.083 mi/min

Corresponding Vertical Miss· 163 mi.

Corresponding Horizontal Miss: 48 mi.

We have already indicated some of the difficulties of the azimuth steering scheme. In Figure 4-17 are plotted, for every trajectory which yielded a cut-off, the magnitude of the error in $\dot{Y}$ at injection versus the powered flight time. Although the points do not form a smooth curve, a definite correlation is obvious between short powered flight time and large errors in $\dot{Y}$ at injection. This tendency is explainable on the basis of the later time (8 35 minutes after lift-off) at which

SUMMARY OF TRAJECTORY SIMULATIONS FOR NON-COPLANAR LUNAR IMPACT MISSION

RERUN NO. 10

I. D	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS														BALLISTIC FLIGHT DATA			GUIDANCE EQUATION PERFORMANCE							
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	LATITUDE (DEG, MIN)	LONGITUDE (DEG, MIN)	△ RA (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	AZIMUTH HEADING (DEG)	X (MI)	Y (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Y}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	IMPACT VELOCITY (FT/SEC)	ERROR IN $\dot{X}$ (MI/MIN)	ERROR IN $\dot{Y}$ TOTAL (MI/MIN)	ERROR IN $\dot{Y}$ AZIMUTH STEERING (MI/MIN)	HORIZONTAL MISS (MI)	VERTICAL MISS (MI)	CHORDAL MISS UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
210	Nominal Trajectory																											
	0	+ 8	0	0	9 486494	30 89754	25 15848	27 53660	36682 05	362 4653	7 708583	95 09568	1736 537	-24 1806	-1.86496	400 6656	-6 59096	-114.806	26 49726	130 4054	11895 04	01731	- 20946	1925651	-83 0638	-39.4641	91 95212	90 4226	17 1155
232	+2 6	+ 6.9	+ 5	+ 11	9 251790	31 04375	24 14837	26 46765	36247 31	377 6657	8.131753	93 74567	1678.361	-18.0014	39.68071	398 5314	- 778745	-104 090	29.06362	132 6512	10683 76	00611	0879199	2107268	41 11692	-11 3089	42 64380	42 58811	3 159217
233	+3 7	+ 8.6	+ 5	- 16	9 052999	31 12914	23 55029	25 81974	35748 97	401 6475	8 875538	92 28761	1648 337	-14.1939	78 08226	395 2330	7.123166	-93.6467	32.91462	135 1436	9226.238	05783	- 314505	0377047	-86 4667	-107 633	138 0634	131 883	22 0816
248	+6	+ 8	0	0	8 817626	31 18755	22 38425	24 59469	35274 94	414.1729	9.353139	89.00279	1578 606	-13 9914	119.2901	391.6336	25.27121	-81.6486	39.44583	139 5015	7411 618	- 12432	2 981843	-1 77248	1265 424	-16 5614	1265 537	1244.302	785 2741
258	- 5	+ 9	+ 5	+ 1	9 505069	30 92166	25 35683	27.73960	36533 35	375 0753	8 031842	94.94988	1753.508	-21.4492	4.452964	399.3688	-4.73432	-113.283	27.10991	130 5516	11583.95	.02373	1697768	1735765	85 00653	-40 9850	94 37100	94 37069	17 69287
259	- 5	+ 9	+ 5	- 1	9 498512	30 89232	25 40392	27 78505	36603 71	370 5379	7.931359	95 08907	1754.509	-23.4137	- 963548	399.8445	-5 59860	-114.492	26.78032	130 3380	11749 21	03113	1179817	1750589	63 11340	-55 4012	83 97980	83 82384	14 44166
260	+4 5	+ 9	+ 5	+ 1	8 961992	31 17991	23 12670	25.37333	35392 82	418 8780	9 364180	90.61589	1627 632	-12.0147	105.0475	392.3870	11.03044	-86.6051	36.91227	137 5913	8036 543	- 07208	912455	- 329916	327 7170	82 15995	337 8591	328 2917	97 40631
261	+4 5	+ 9	+ 5	- 1	8 963177	31 16827	23 20784	25 45477	35479 59	414.8518	9.237313	91.08975	1631 109	-12 6014	99.31345	393.1550	14.07738	-88 2206	35.78754	136 9409	8343 778	- 02409	434757	- 205937	167 9211	24 72269	169 7313	166 2829	28 65988
262	+4 5	+ 7	+ 5	- 1	9 030866	31 12668	23 31805	25 58194	35868 05	390 9276	8.591491	92.46828	1629.999	-15.2275	73.95726	396.4449	5.00400	-94.5369	32.05908	134 8869	9521 718	.11285	- 66561	0906537	-195 980	-214 754	290 7370	278 910	95 5262
263	+4.5	+7	+ 5	+ 1	9 053897	31 14094	23 37167	25 64134	35828 74	396.2131	8.656171	92.41432	1635.501	-13.9459	77.43713	396.0273	5.610244	-94.3281	32.25933	134 9449	9451 200	09974	- 583549	0828563	-170 415	-188 346	253 999	243 480	73 7920
230	+3 7	+10	0	+ 2	9 036846	31 18842	23 37665	25 64204	35463 23	419.9202	9.310483	91.22098	1644.260	-10.3963	99.47801	392.8464	13.86467	-88 7775	35 73456	136 7469	8359 392	- 00773	139747	- 391028	53 80003	8 263994	54 43103	53 31198	3 037643
235	+4.8	+ 7 4	+ 5	- 16	9 006433	31 14309	23 30986	25 56763	35784 71	398.3214	8.714169	92.25796	1631.844	-14.0590	81 19039	395.7281	6.427629	-93 3685	32.65584	135 2086	9315 008	10098	- 590005	0503828	-169 216	-188 744	253 4932	242 637	71 5656
236	+1 4	+ 8.6	+ 5	- 2	9 311800	31 00735	24 64104	26 97536	36294 47	381.0329	8.204659	94.11764	1710 104	-18.5808	29.87022	398.3774	-1.56206	-106.758	28.59971	131 9850	10885 39	- 00691	2709093	1929253	113 4539	16 31113	114 6205	113 4060	22 46590
239	+7 0	+ 6.9	+ 5	- 11	8 744957	31 17147	22 11876	24 31098	35267 79	409.1452	9 282000	88.67611	1559.587	-16.0468	121 2113	391 7248	26.47482	-80.4221	40.01921	140 0478	7282 106	- 16245	3 024778	- 744355	1228 650	37 07279	1229 213	1204 121	737 3698
240	+5 9	+ 8 6	+ 5	- 2	8 797101	31 16869	22 48845	24 69374	35217 66	417.6426	9.474821	88.81465	1585.949	-15 1339	120.1587	390.9521	26.84923	-81.2168	40.38299	139 9090	7202 428	- 16541	2 69281	- 911344	1069 550	64 05043	1071 469	1046 757	592 6184
242	+4 8	+10.0	+ 5	+ 06	8 848739	31 17440	22 68566	24 90390	35066 05	429.0208	9 787208	88.02955	1602.866	-14.0631	125.7705	389.0278	32.83076	-79.7758	43.02927	140 9888		NO SOLUTION		- 560095	858 7577	-184 177	878 2874	870 2165	404 8842

Figure 4-16
(No. 1 of 3 sheets)

SUMMARY OF TRAJECTORY SIMULATIONS FOR NON-COPLANAR LUNAR IMPACT MISSION

RERUN NO. 10 (Cont'd)

I D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS														BALLISTIC FLIGHT DATA			GUIDANCE EQUATION PERFORMANCE							
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	LATITUDE (DEG, MIN)	LONGITUDE (DEG, MIN)	△ RA (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	AZIMUTH HEADING (DEG)	X (MI)	Y (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Y}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	IMPACT VELOCITY (FT/SEC)	ERROR IN $\dot{X}$ (MI/MIN)	ERROR IN $\dot{Y}$ TOTAL (MI/MIN)	ERROR IN $\dot{Y}$ AZIMUTH STEERING (MI/MIN)	HORIZONTAL MISS (MI)	VERTICAL MISS (MI)	CHORDAL MISS UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
246	- 0 8	+9 6	+ 5	- 16	9 503311	30 89540	25 51780	27 90139	36576 85	374 1240	8 014685	95 12798	1762 827	-22 6244	- 730984	399 5217	-5. 43247	-114 518	26 85982	130 2766	11709 24	03669	032135	.1670369	30 55647	-68 0793	74 62239	73 77550	11 18739
252	+ 7	+6 0	0	+ 2	8 794394	31 19446	22 08157	24 28619	35379 72	403 9422	9 086343	89 43898	1556 152	-14 3499	116 9599	393 0154	21 23777	-82 0117	38 18291	139 1199	7713 737	- 03590	1 83365	- 764038	810 1791	-63 7449	812 6341	803 2616	424 5453
253	- 1 9	+8.6	0	- 11	9 658491	30 73345	26 01081	28 43452	37048 42	349 5365	7 403875	96 12184	1784 977	-31 9032	-37 7343	402 3040	-10 8344	-123 608	24 79548	128 7334	12837 77	04707	- 655569	.2412715	-270 282	-112 819	292 8832	289 065	152 499
254	+ 1 4	+7.4	0	+ .16	9 362064	31 00207	24 53583	26 88275	36420 96	371 5693	7 946071	94 31363	1700 896	-19 3482	23 51313	399 4160	-3 31700	-108 388	27 95185	131 7297	11180 81	- 01100	- 12059	2105682	-57 5065	18 89318	60 53063	60 4900	6 92678
255	+ 5 9	+8 6	0	- 16	8 79200b	31 18403	22 35899	24.56301	35216 94	415 9763	9 438140	88 73221	1577 340	-14 3627	121 7119	391 0429	26 96279	-80 7001	40 54764	140 0333	7167 915	- 13191	2 25408	- 845622	902 1005	45.05891	903 2268	882 7274	452 7256
268	- 0 8	+10.0	+ 5	+ .06	9 499339	30 95135	25 43150	27 81284	36415 67	364 2622	8 258379	94 82054	1761 415	-18 8892	11 01088	398 4216	-3 49339	-111 760	27 64732	130 7657	11323 30	02198	12738	1600226	66 42767	-39 2936	77 17917	77 16743	11 45734
270	-3 0	+6 4	0	+ 16	9 044210	31 15950	23 16614	25 43338	35867 27	391 5264	8 529319	92 45255	1621 156	-13 2751	78 17232	396 5496	4 600648	-94 0793	32 04284	134 9835	9529 812	.15882	-1 31126	0946210	-435 397	-296 122	526 5542	508 799	263 787
272	- 1 9	+10 0	0	- 11	9 608493	30 83708	25 86568	28.27438	36746 77	367 5926	7 856921	95 66714	1782 469	-25 0790	-16 7194	400 3542	-7 98438	-118 420	26 00699	129 4960	12155 16	06068	- 72436	1767993	-292 719	-136 037	322 7866	317 693	166 196
251	+2	+8	0	0	9 264034	31 04460	24 23543	26 55778	36209 80	380.9869	8 238433	93 79968	1685 024	-17 5737	40.48378	398 1726	-.826178	-103.776	29.25849	132 6640	10601 64	01586	3886902	2020548	-157 936	-33 7547	161 5297	159 330	40 8971
*249	+7.0	+8	0	0	8.629131	31 16405	21.48538	23.64857	34058 59	416.5755	9.752645	89.43333	1521.786	-18.4346	143.3762	386 5884	18 47210	-72 18993					SEE NOTE BELOW						
*267	+7 0	+7.8	0	+.2	8 627693	31.16690	21.37861	23.54144	35,237.6	423 465	9.818016	89.40109	1515 591	-18.4873	146 8170	385 5249	18 24636	-70 89942					SEE NOTE BELOW						

* FOR THESE RUNS, INJECTION CONDITIONS, AS PROVIDED BY THE GUIDANCE SYSTEM, LED TO A BALLISTIC TRAJECTORY WHICH DID NOT REACH THE RADIUS OF THE MOON'S ORBIT.

Figure 4-16
(No 2 of 3 sheets)

SUMMARY OF TRAJECTORY SIMULATIONS FOR NON-COPLANAR LUNAR IMPACT MISSION

RERUN NO. 10 (Cont'd)

I. D.	OFF NOMINAL DEVIATIONS				INJECTION CONDITIONS														BALLISTIC FLIGHT DATA			GUIDANCE EQUATION PERFORMANCE							
RUN NUMBER	FLOW RATE (PERCENT)	SPECIFIC THRUST (PERCENT)	LIFT OFF DELAY (MIN)	PITCH OFFSET (DEG)	TIME (MIN)	LATITUDE (DEG, MIN)	LONGITUDE (DEG, MIN)	△ RA (DEG)	VELOCITY (FT/SEC)	ALTITUDE (MI)	PATH ANGLE (DEG)	AZIMUTH HEADING (DEG)	X (MI)	Y (MI)	Z (MI)	$\dot{X}$ (MI/MIN)	$\dot{Y}$ (MI/MIN)	$\dot{Z}$ (MI/MIN)	TIME OF FLIGHT (HRS)	SWEEP ANGLE (DEG)	IMPACT VELOCITY (FT/SEC)	ERROR IN $\dot{X}$ (MI/MIN)	ERROR IN $\dot{Y}$ TOTAL (MI/MIN)	ERROR IN $\dot{Y}$ AZIMUTH STEERING (MI/MIN)	HORIZONTAL MISS (MI)	VERTICAL MISS (MI)	CHORDAL MISS UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH UNFOCUSED (MI)	DISTANCE OF CLOSEST APPROACH FOCUSED (MI)
231	-1 9	+ 6 9	0	+ 2	9 680500	30 69142	25 67252	28.09927	36458	330 475	6.807700	96 60248	1757.696	-36 6298	-46.3587	395 0681	-15 3930	-123.783										
213	-3	+ 8	0	0	9.785000	30.59841	26 34184	28 79479	36,955	331 263	6 916355	96 81363	1799 163	-39 9090	-64 4166	399 3858	-14.5629	-128 944										
256	- 5	+ 7	+.5	-.1	9 547500	30 77798	25 39068	27.78408	36,644	343 114	7 200709	95 81839	1743 912	-31 8310	-26 3237	398 6652	-10 9499	-119.774										
257	- 5	+ 7	+.5	+.1	9 547500	30.81067	25 30658	27.69999	36,523	347 323	7 299005	95 68152	1740.504	-29.8223	-20 1964	397.7237	-10.2217	-118 176										
241	-1.9	+ 6	0	-.06	9 680500	30 59884	25.64584	28.07259	36,451	311 361	6 339272	96 98634	1748 776	-43.4796	-63 3772	393 8814	-18.3050	-126 884										
244	+0 3	+ 6.4	0	+ 02	9 471500	30.85040	24.87175	27.24611	36,379	342 717	7 158657	95 50668	1711.711	-28 9543	-12.6129	396 6514	-10.5999	-115 967										
237	-1 9	+7 8	+ 5	-.06	9 680500	30 68609	26.01866	28 44541	36,929	341 423	7 195042	96 31170	1782 506	-35.3277	-45 5541	400.4810	-12.1734	-124.765										
247	-3 0	+ 6	+ 5	+ 11	9 785000	30.49661	26 06095	28 51389	36,385	305 275	6 205755	97 34771	1772.500	-48 7240	-80.8679	391.9480	-19 2744	-130.236										
265	-1.9	+7 8	0	-.06	9 680500	30.69275	25.93839	28 36513	36,958	339 046	7 127922	96 33659	1776 786	-35 2323	-45 5625	400 7914	-12 6628	-130.847										
266	-3.0	+7.4	+.5	+ 06	9 785000	30 63621	26 50128	28.95423	37,113	344 597	7.307250	96.54887	1813.865	-36 3421	-56.4973	401 7617	-12.0605	-127 694										

* FOR THE RUNS ON THIS SHEET, FUEL WAS EXHAUSTED BEFORE GUIDANCE SYSTEM INDICATED CUT-OFF, THEREFORE, INJECTION CONDITIONS LISTED ARE AT BURN-OUT.

Figure 4-16
(No 3 of 3 Sheets)

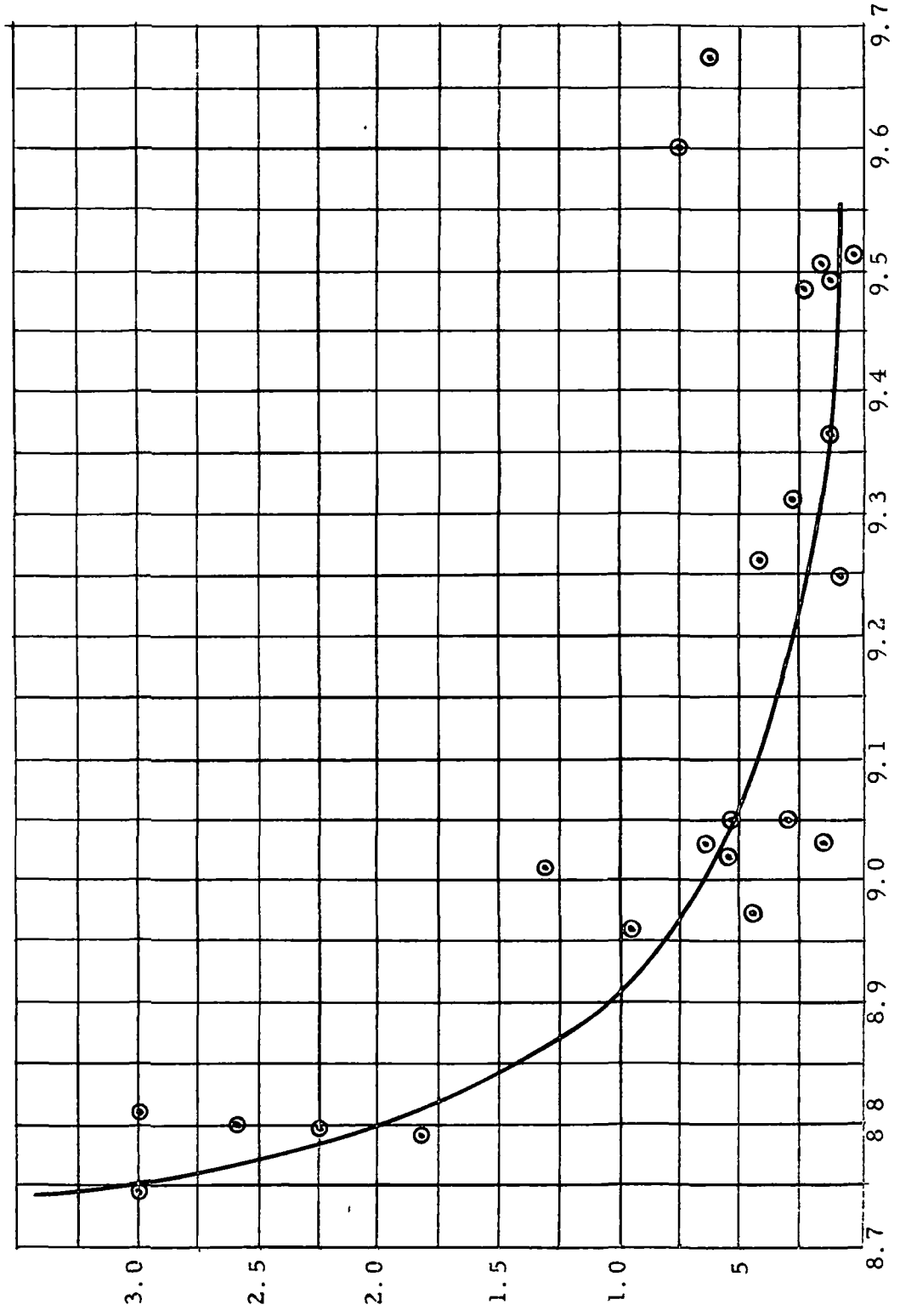

Figure 4-17 MAGNITUDE OF ERROR IN $\dot{y}$ AT INJECTION VS. TIME FROM LIFT-OFF TO CUT-OFF

computed azimuth steering takes control. Thus, on runs with short flight time, there is insufficient time for azimuth steering to take out the errors in $\dot{Y}$. This seems to show great promise for the idea mentioned earlier of computing the switch time while airborne rather than using a fixed, predetermined value.

We summarize below the azimuth steering performance for all 24 runs which yielded cutoff points.

RMS error in $\dot{Y}$: 1 275 mi/min

Corresponding Vertical Miss. 8.7 mi.

Corresponding Horizontal Miss: 548 mi.

If we eliminate the 3 runs which yielded the largest errors in $\dot{Y}$ (runs no. 239, 248, 240), then the azimuth steering performance summary for the remaining 21 points is:

RMS error in $\dot{Y}$: 0 812 mi/min

Corresponding Vertical Miss: 5.6 mi.

Corresponding Horizontal Miss: 348 mi

As might be expected, short flight times (and therefore large errors in $\dot{Y}$) are associated with high mass flow rates, since under this off-nominal condition velocity builds up faster This can be seen by comparing mass flow rates for the five runs with the largest $\dot{Y}$ errors with the mass flow rate for the 5 runs with the smallest $\dot{Y}$ error:

5 Largest $\dot{Y}$ Errors vs. Flow Rate Deviation

Run No.	Error in $\dot{Y}$ (mi/min)	Flow Rate Deviation (%)
239	3.025	+7.0
248	2.982	+6.0
240	2.693	+5.9
255	2.254	+5.9
252	1.834	+7.0

5 Smallest $\dot{Y}$ Errors vs. Flow Rate Deviation

Run No	Error in $\dot{Y}$ (mi/min)	Flow Rate Deviation (%)
246	032	- .8
232	088	+2.6
259	.118	- .5
254	-.121	+1.4
268	.127	- .8

The correlation between high off-nominal mass flow rate and large azimuth steering errors is clearly evident from the above tabulations.

On runs no 249 and 267, the cutoff conditions indicated by the guidance system resulted in a ballistic trajectory which did not reach the radius of the moon's orbit. The reason for this behavior is not clear, and an explanation will have to await more careful investigation for which there was no time on the present study

On sheet no. 3 of Figure 4-16 are shown trajectories on which fuel was completely exhausted before the guidance system indicated a cutoff. It is evident that these trajectories all had rather low flow rates and specific impulse (recalling that the specific impulse deviation for the nominal trajectory, run 210, is +8%).

In order to arrive at an overall judgment of the capability of the guidance system approach, let us concentrate on the vertical miss, since it is clear that horizontal miss performance will improve greatly with simple changes to the azimuth steering scheme. Perfectly satisfactory vertical error performance has been obtained over the range of mass flow rates from 7% (runs 252, 239) to about -2% (runs 253, 272). With a more judicious choice of the nominal trajectory, this range can be symmetrized to ±4.5%, which is compatible with variations expected of single thrust chamber engines. We shall comment on cluster engines later in this discussion.

SECTION 5

OVERALL CONCLUSIONS AND DISCUSSION

In the preceding sections of this volume, we have described the formulation of the guidance system concept and have presented simulation results to evaluate the concept. Before closing this volume, let us see what general conclusions and recommendations can be reached as a result of the work which has been presented.

Generally speaking, it is clear that for vehicles with single thrust chamber engines the guidance concept presented appears entirely feasible for space mission injection guidance (of which the lunar mission examined may be considered typical). For such vehicles, the expected variation in mass flow rate is within the ±4.5% which is allowable with the guidance concept presented. For vehicles with clustered engines, however, the situation is somewhat different. These engines are designed with a certain amount of redundancy for reliability, so that payload-lifting capability is insured in the event of an engine failure. However, this implies that for a cluster of 8 engines a drop in thrust level of 12-1/2% for a significant portion of the flight is entirely possible from an operational viewpoint, should an engine fail. Thus it is felt that additional systems analysis studies are imperative in order to arrive at an injection guidance system concept which is not critically dependent on the behavior of the vehicle's propulsion system. One might hope to achieve this through further study of the present guidance concept It is felt, however, that order of magnitude improvements will probably not be achieved in this way for two primary reasons:

. . .The final non-coplanar results presented were for a rather high velocity trajectory (nominal ballistic flight time of 26.5 hours). Certainly, lower velocity trajectories are important when maximum payload on the moon is desired, or when minimum residual velocity at the moon is desired to ease braking for a soft landing. As demonstrated in the coplanar simulation results (Section 4.4.2), low velocity trajectories have higher sensitivities of miss to guidance errors.

....The guidance expansions already have a rather large number of terms (24 terms each in the cutoff and azimuth steering expansions). Therefore, it is not expected that great gains can be made by further increase in the number of terms.

It is recommended, therefore, that a guidance concept which does not place such heavy emphasis on staying close to a nominal trajectory be developed. For example, it was mentioned in Section 3.6 that Arma has under development a flexible airborne digital computer which has the capability of computing the analytically correct expressions for $\dot{X}_r$ and $\dot{Y}_r$. Such a technique, or a similar one, might show great promise as the basis of an injection guidance system concept for space missions.

A conclusion of the study (Section 2) was that the pitch control channel should not be committed to the requirement of impacting the moon but should be utilized to satisfy other mission requirements. The present study has demonstrated that the impact conditions can in fact be satisfied using only thrust termination and azimuth steering guidance. It is, therefore, now appropriate to conduct further studies to see how the pitch channel can be used to satisfy other important mission requirements. An example is control of the injection conditions to insure that residual velocity at the moon will be within sufficiently narrow limits so that large amounts of excess braking fuel will not be required for soft landings.

This study has been limited to the lunar impact mission. It is clear that with a minimum effort, the results could be expended to gain an understanding of injection guidance for offset shots such as are required to establish an orbit around the moon. It is recommended that such a study be undertaken.

It was mentioned that on certain non-coplanar trajectories with low mass flow rate and low specific impulse, the rocket fuel is exhausted before the guidance system indicates a cut-off (see Sheet No 3 of Figure 4-16). This situation is more complex than might appear, since in general, sufficient velocity has been achieved on these runs to reach the radius of the moon's orbit. However, if insufficient care is taken in designing the nominal trajectory, extremely high velocity may be required under off-nominal conditions to hit the moon. An extreme example of such a situation occurs when the extended tangent to the powered flight trajectory passes through the moon. At this moment, infinite velocity is required for a lunar impact. A study should be conducted to arrive at a method of selecting nominal trajectories which avoids these difficulties.

Although for vehicles with clustered engines, the polynomial expansion guidance technique has some problems, it appears to show great promise for vehicles with single thrust chamber engines. Therefore, studies should be performed to clear up some problems remaining with this approach. Among these are the following:

....It is quite certain that with a moderate effort the azimuth steering performance could be greatly improved.

....Investigations should be conducted to explore the feasibility of the polynomial expansion technique for low velocity non-coplanar trajectories.

....The operational concept for dealing with delayed launch involves the resetting of launch conditions at predetermined instants of time. Further studies of this technique should be conducted to determine the allowable reset interval. (The study established this interval as greater than 30 seconds, but the upper limit was not determined.)

....The mathematical form of the guidance expansions should be optimized to insure that all necessary terms are included and all excess terms are excluded.

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APPENDIX

FORMULATION OF THE NAVIGATION PORTION OF THE GUIDANCE SYSTEM

In the Figure 3-6, it can be seen that the navigation variables X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$, t which form the inputs to all guidance computations are generated by the navigation portion of the system. In this portion of the system, accelerometers mounted on the inertial platform furnish sensed acceleration data to the navigation portion of the computer which operates on this data to yield position and velocity outputs.

In the discussion concerning the formulation and evaluation of the guidance system concept which was presented in this volume, no consideration was given to the navigation portion of the system. The reason for this omission is that most of the navigation errors arise because of non-ideal characteristics of the hardware, which are discussed in detail in Volume IV of this report.

There is, however, one aspect of the inertial navigation process which should properly be included in a discussion of the system concept. This is the gravity feedback computation which is considered in this Appendix.

Fundamentally, the i^{th} component of velocity and position are computed by two successive integrations of the i^{th} component of total acceleration ($i = X, Y, Z$).

It is well known, however, that the accelerometers of an inertial guidance system are capable of sensing only the non-gravitational accelerations (e.g., thrust) of the vehicle. Therefore, the effect of gravitational acceleration on the vehicle must be taken into account by a feedback computation method. In this method, the inertial navigator position outputs are used to compute the gravitational accelerations, which are then added to the outputs of the accelerometers to yield the total acceleration acting on the vehicle.

The subject of this Appendix is the form of the equations for computing gravity accelerations from position data which is appropriate for injection guidance of space vehicles.

These equations must satisfy two requirements:

....They must be sufficiently accurate so that injection position and velocity errors from this source are not excessive.

....Their form must be as simple as possible in order to minimize the computing load on the missile borne digital computer.

Other investigations have solved the problem under discussion for the case of terrestrial missions. Gravity equations were derived which yielded negligible position and velocity errors at engine cut-off for various "non-space" applications.

Therefore, in studying injection guidance for space vehicles, it appeared reasonable, as a first trial, to investigate the applicability of the terrestrial gravity computations to the more advanced space missions. Minor variations from the terrestrial mission computation method, were also investigated. The conclusion drawn from these investigations was that for space missions a new gravity computation method was required. This is because the increased time of powered flight and the greater downrange distance to the injection point lead to excessive position and velocity errors at injection when the terrestrial approach is used.

A new approach to the gravity computation problem has been developed and has been found to yield negligible position and velocity errors at injection. These errors are small enough so that they have negligible effect on the guidance system evaluation results presented in Section 4.4.

The remainder of this Appendix presents information which documents the above conclusions in greater detail.

We assume for illustrative purposes a two-dimensional lunar mission with due east firing from the equator. The coordinate system used is stabilized in inertial space, with X eastward and Z vertical at the launch point. It is easily seen that

$$g_x = \frac{KX}{r^3} = KXr^{-3} \qquad \text{(A-1)}$$

$$g_z = \frac{K(Z+r_e)}{r^3} = K(Z+r_e)r^{-3} \qquad \text{(A-2)}$$

where

r_e is the radius of the Earth

g_x is X component of gravity acceleration

g_z is Z component of gravity acceleration

r is vehicle distance from Earth's center.

(Note: In this analysis, we ignore the effects of Earth's oblateness and the slight misalignment of the Z axis from the Earth's radius vector which would occur at a non-equatorial launch point due to centrifugal effects. These effects will be covered later in the discussion

In the terrestrial mission approach, r^{-3} is represented by a polynomial in X and Z :

$$r^{-3} \approx K_0 + K_1(X-X_0) + K_2(Z-Z_0) + K_3(X-X_0)^2 + K_4(Z-Z_0)^2 + K_5(X-X_0)(Z-Z_0) + K_6(X-X_0)^2(Z-Z_0) \qquad \text{(A-3)}$$

in which the K's are computed by a Taylor series expansion about a fixed point at specified values of X_o and Y_o.

Figure A-1 shows the results of using this expansion for the lunar mission with X_o = 200 miles, Y_o = 100 miles.

Differences between true and calculated gravitational acceleration components are plotted as a function of downrange distance (X) for a nominal trajectory (low velocity coplanar run no 15).

A table is given of errors in position and velocity at injection for the nominal trajectory. It can be seen that the velocity errors are far too large. It is important to note that while inappropriate for space missions, the gravity error curves do show negligible errors for downrange distances appropriate for terrestrial missions.

Several variations of equation (A-3) were attempted. A least squares method of fitting constants, and different choices for the position of the expansion point X_o, Z_o were tried. The resulting errors at cutoff were still too large however. At this point, it was decided to drop the expansion for r^{-3} in the form given in equation (A-3). It was concluded that the excursions of X and Z from the point about which the expansion is made are too great to achieve good accuracy with a reasonable number of terms. Hence the following approach was taken: First r^{-2} is computed from

$$r^2 = X^2 + (Z + r_e)^2 = Q \qquad \text{(A-4)}$$

It follows then that

$$r^{-3} = Q^{-\frac{3}{2}} \qquad \text{(A-5)}$$

We now approximate (A-5) by

$$r^{-3} \approx C_0 + C_1 Q + C_2 Q^2 + C_3 Q^3 + C_4 Q^4 \qquad \text{(A-6)}$$

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As before, g_X and g_Z are computed by equations (A-1) and (A-2), where the C's are derived by Taylor Series expansion about some predetermined value of Q. It is to be expected that an expansion in terms of $Q\ (=r^{-2})$ should show improved results because the total variation in r^2 is quite small.

The results of this approach are shown in the table below. The errors at the acceleration level are too small to be seen to the scale used for Figure A-1. The largest error in g_X is $-8.59 \times 10^{-5} g$ and in g_Z the largest error is $-10.8 \times 10^{-5} g$. In velocity and position, both the absolute magnitude and the spread have been reduced to negligible values.

The table below summarizes the accuracy of the new gravity computation. The nominal trajectory is the low velocity coplanar run no. 15.

Trajectory	Errors at Injection			
	$\Delta\dot{X}$ (ft/sec)	$\Delta\dot{Z}$ (ft/sec)	ΔX (mi)	ΔZ (mi)
Nominal	-.009	-.012	-.00009	-.002
Mass Flow +5%	-.053	-.126	-.0005	-.0004
Mass Flow -5%	-.0006	+.016	-.000005	+.00018

The changes necessary to include the third dimension in the above analysis are obvious. Equations (A-1) and (A-2) are generalized to

$$\left.\begin{aligned} g_x &= \frac{K_X}{r^3} = K_X Q^{-3/2} \\ g_y &= \frac{K_Y}{r^3} = K_Y Q^{-3/2} \\ g_z &= \frac{K_Z}{r^3} = K(Z+r_e) Q^{-3/2} \end{aligned}\right\} \qquad (A-7)$$

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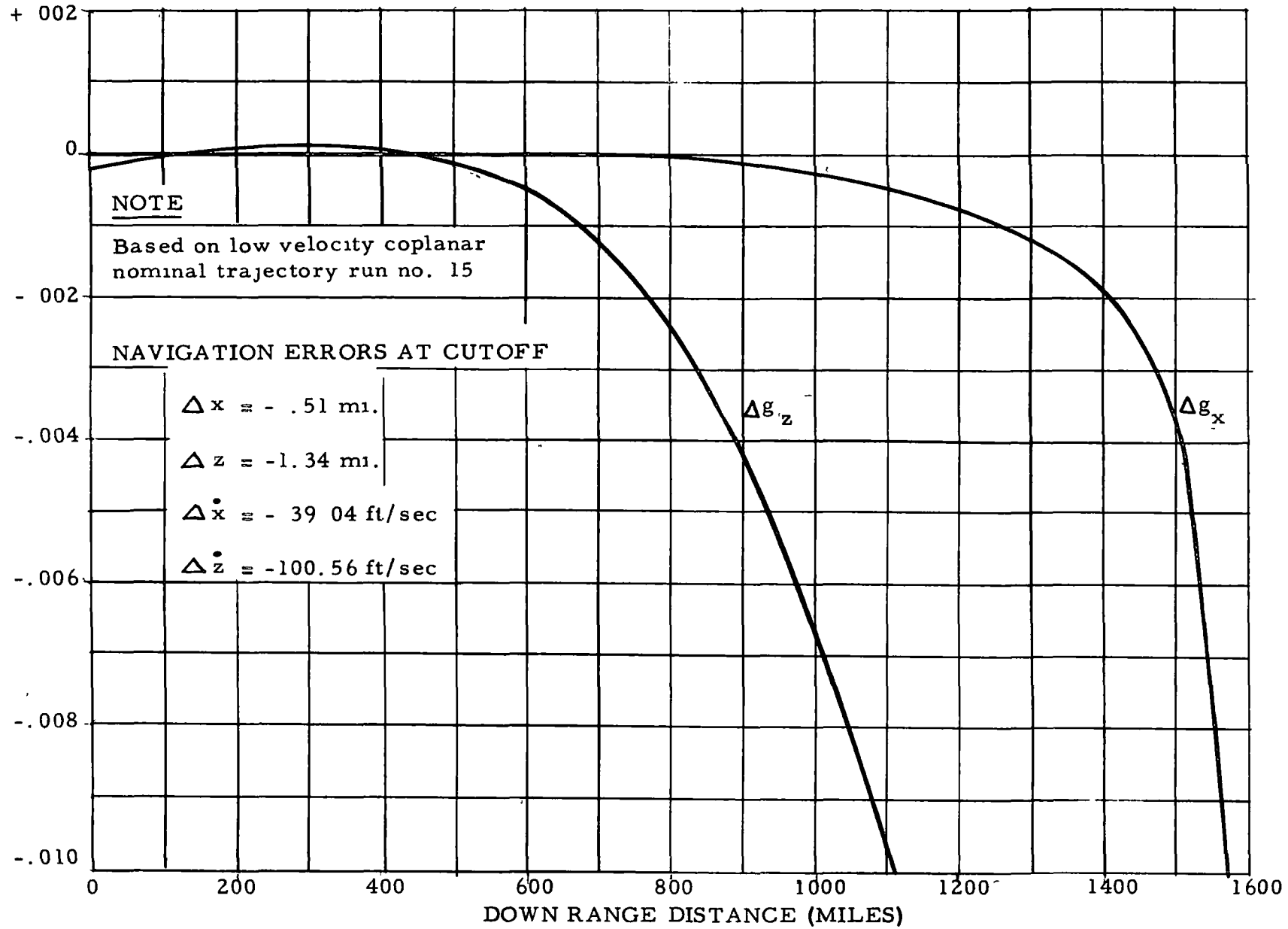

Figure A-1 LUNAR MISSION ASCENT PHASE NAVIGATION ERRORS DUE TO GRAVITY COMPUTATION APPROXIMATIONS USING GRAVITY COMPUTATION APPROPRIATE FOR TERRESTRIAL MISSIONS

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where

$$Q = x^2 + y^2 + (z + r_e)^2 \tag{A-8}$$

and $Q^{-3/2}$ is still approximated by

$$Q^{-3/2} = C_0 + C_1 Q + C_2 Q^2 + C_3 Q^3 + C_4 Q^4 \tag{A-9}$$

There are two effects which have not been included in the analysis thus far:

....The effect of Earth oblateness

....The effect of centrifugal force due to Earth spin on the orientation of the inertial coordinate system (caused by the fact that pre-launch alignment is with respect to the plumb-bob vertical, which is affected by centrifugal force).

These effects are included by making the following modifications:

Equation (A-7) becomes

$$\left.\begin{aligned} g_x &= K(x + \delta_x)Q'^{-3/2} \\ g_y &= K(y + \delta_y)Q'^{-3/2} \\ g_z &= K(z + \delta_z + r_e)Q'^{-3/2} \end{aligned}\right\} \tag{A-10}$$

where δ_x, δ_y, δ_z are small constants which are functions of launch latitude and azimuth orientation of the platform. Q' is computed using a slight modification of equation (A-8):

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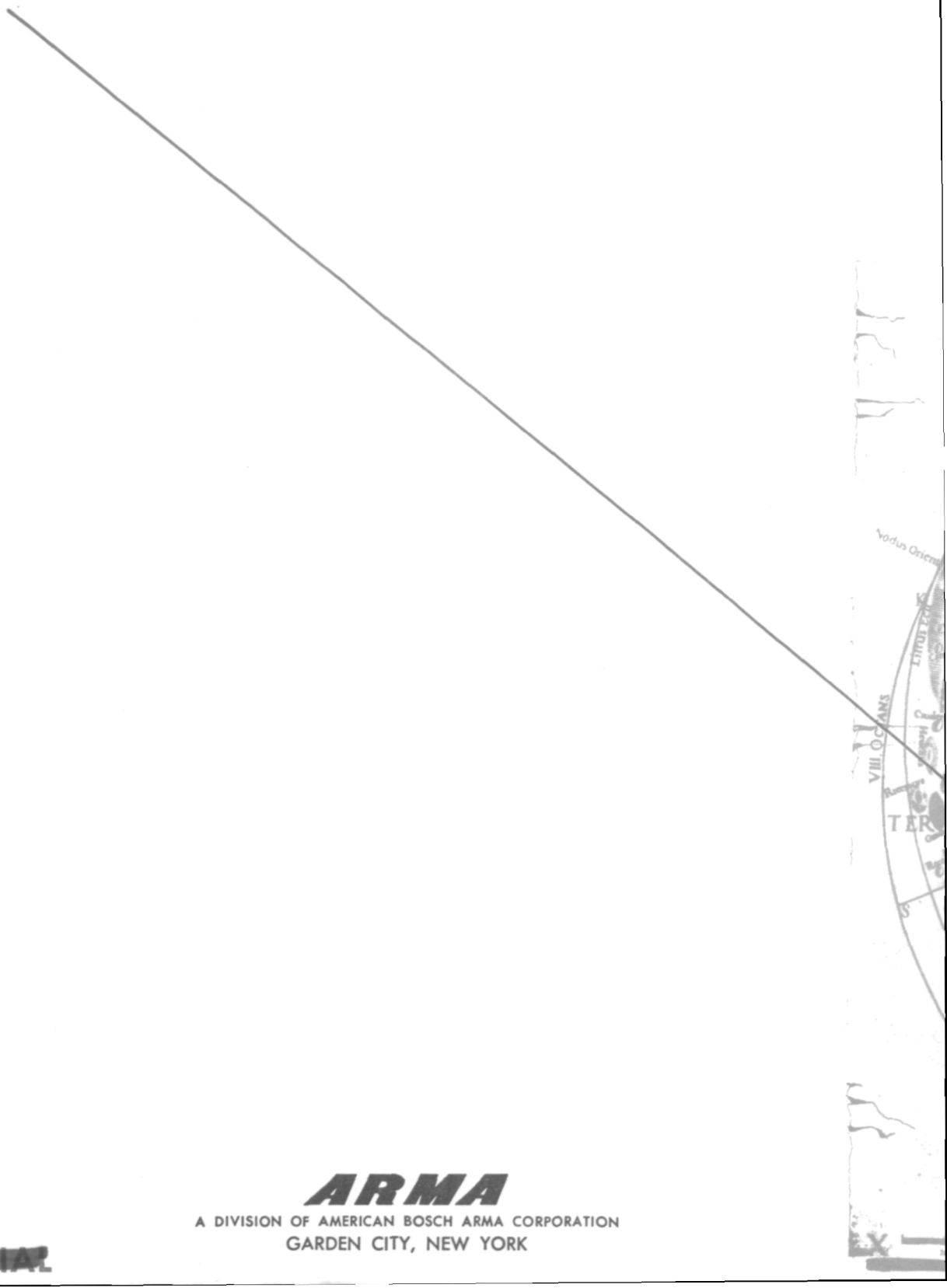

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